Logicism and the Philosophy of Mathematics

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Outline

1. History of logicism
   - Frege's logicism
   - Russell's logicism
   - Logicism of Vienna Circle - Rudolf Carnap
   - Neologicism
2. Logicism and logical pluralism

History of logicism

- Introduced and developed by Frege.
  - *Begriffsschrift*, 1879
  - *Die Grundlagen der Arithmetik*, 1884
  - *Grundgesetze der Airthmetik*, 1893, 1903
- Russell's type theory and *Principia Mathematica* (1910).
- Logicism of Vienna Circle - Carnap, Ayer.
- Revival of logicism.
  - Neofregeanism - Crispin Wright, Bob Hale
  - Epistemic logicism - Bernard Linsky, Edward Zalta

Frege's logicism

- Introduced in *Begriffsschrift* in 1879.
- Further developed in *Die Grundlagen der Arithmetic* in 1884.
  - Main goal: derivation of arithmetic by logical means alone
  - Outlining the proof of the derivation.
- Formal system introduced in *Grundgesetze der Arithmetik*.

Frege's logicism - Grundgesetze

- Logical axioms, rules of inference, proof system.
- Definition of numbers.
- Derivation of a version of Peano axioms.
- System is not consistent.

Russell's logicism

- Reformulation of logic - preventing semantical and logical paradoxes.
- Theory of types (1903, 1908).
- *Principia Mathematica* (Whitehead, Russell 1910,1912,1913).

Russell's logicism - *Principia Mathematica*

- Logical foundation for mathematics.
- Solution for paradoxes.
- Ramified type theory and no-class theory.
- Axioms of Reducibility, Choice, and Infinity.
- Not a system of logic

Carnap's logicism

- Metaphysical questions are meaningless.
- Scientific discourse is meaningful only in a predetermined *linguistic framework*.
- Mathematics can be analytical.

Carnap's logicism

- *Principle of tolerance* - linguistic frameworks are adopted on a pragmatic ground.
- Mathematics is analytic in some linguistic framework if it can be logically developed in that framework.
- Mathematical entities are "framework entities".

Neofregeanism

- The propositions of arithmetic are analytic and mathematical objects exist independently of our mind.
- Hume's principle.

$$\forall F \forall G (NF = NG \leftrightarrow F \approx G)$$

- Second-order logic + Hume's Principle is enough for the derivation of arithmetic (Wright 1983).
- Second-order logic + Hume's Principle is consistent (Boolos 1987).

Neofregeanism - questions and problems

- Is Hume's Principle analytical?
- Is second-order logic "true" logic?
- *Caesar's problem*.
- Impredicativity.
- Bad company objection.
- Embarrassment of riches.

Logicism and logical pluralism

- Logicism adheres to the existence of a single universal logical system.
- How to reconcile the existence of many logical systems with the universality of logicism?

1. Set some restrictions.
2. **Accept the plurality**.

Logicism and logical pluralism

- Carnap's principle of tolerance.
- Set theory as explication - there is no true notion of set.
- There is no one true logic.
- There is no one universal logical system.

Logicism and logical pluralism

- Every "good enough" formal system describes only some part of the logical universe.
- The system is "good enough" if it is (relatively) consistent or its consistency is undoubtful (Peano Arithmetic, ZFC, NBG).

Advantages

- An abundance of logical systems is not a problem.
- Ontology of mathematics is not important - applicable from platonism to nominalism.
- Numbers are given based on a model we use.
- In accordance with mathematical practice.

Disadvantages

- Weak version of logicism.
- Vague notion of "good enough".
- Mathematics is not analytical.
- Every system must be based on some stronger system.

Summary

- New view on logicism.
- Similar to Carnap's philosophy of mathematics.
- Successfully solves some philosophical problems.
- Relativistic in nature.