14TH INTERNATIONAL CONFERENCE

Logic and Applications

LAP 2025

September 24 - 28, 2025
Dubrovnik, Croatia

Book of Abstracts

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Book of Abstracts of the 14th International Conference on Logic and Applications - LAP 2025, held as a hybrid meeting hosted by the Inter University Center Dubrovnik, Croatia, September 24 - 28, 2025.

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LAP 2025 Web site: `https://lap.math.hr/LAP2025/` Maintained by Marko Horvat, University of Zagreb, and Simona Prokić, University of Novi Sad.

Contents

New Foundations and Strongly Cantorian Sets

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Keywords:
New Foundations, Strongly Cantorian sets, Stratification

New Foundations is a set theory, introduced by Quine in 1937, axiomatized by the axiom of extensionality and the axiom of stratified comprehension. Its main feature is that it allows the existence of very large sets, such as the universal set, the set of all ordinals, the set of all cardinals, etc. The usual set-theoretic paradoxes are then blocked by limiting set-defining formulas only to stratified formulas (well-built formulas in the simple theory of types). As a consequence, the global singleton function $x \mapsto \{x\}$ cannot exist. However, restricting the said function to certain sets is consistent with the theory, and such sets are called strongly Cantorian sets.

In this talk, we will survey the properties of strongly Cantorian sets and show their significance for the theory. In addition, we will explore their connection with the notion of stratification.

Acknowledgments. This work is supported by Croatian Science Foundation under the project HRZZ-IP-2024-05-3882.

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Decidability of Bernays–Schönfinkel Class of Gödel Logics

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Keywords:
Goedel Logics, Prenex Fragment, Bernays–Schönfinkel class, Decidability.

It is well-known that any first-order formula in classical logic is logically equivalent to one in prenex form. Quantifier prefixes define fragments of first-order logic, characterized by prenex formulas containing specific prefixes. Early research identified some such fragments as having decidable satisfiability and validity, while others were undecidable.

In 1928, P. Bernays and M. Schönfinkel proved the decidability for the class of function-free sentences with prefixes $\exists\bar{x}\forall\bar{y}A(\bar{x},\bar{y})$ (satisfiability) and $\forall\bar{x}\exists\bar{y}A(\bar{x},\bar{y})$ (validity) (specifically, the set of sentences that, when written in prenex normal form, have a prefix containing quantifiers and the matrix without function symbols) [5].

In this talk we examine the decidability of the Bernays–Schönfinkel class within Gödel logics. The argument relies on the availability of Skolemization for prenex Gödel logics for validity, and general properties of prenex formulas for satisfiability. Notably, Gödel logics differ from classical logic in that validity and satisfiability are not dual.

Definition (Gödel logics): First-order Gödel logics are a family of many-valued logics where the truth values set (known also as *Gödel set*) V is closed subset of the full $[0,1]$ interval that includes both 0 and 1 given by the following

evaluation function $\mathcal{I}$ on V

$$
\begin{array}{ll}
(1) & \mathcal{I}(\bot) = 0 \\
(2) & \mathcal{I}(A \wedge B) = min\{\mathcal{I}(A), \mathcal{I}(B)\} \\
(3) & \mathcal{I}(A \vee B) = max\{\mathcal{I}(A), \mathcal{I}(B)\} \\
(4) & \mathcal{I}(A \supset B) = \begin{cases} \mathcal{I}(B) & \text{if } \mathcal{I}(A) > \mathcal{I}(B), \\ 1 & \text{if } \mathcal{I}(A) \leq \mathcal{I}(B). \end{cases} \\
(5) & \mathcal{I}(\forall x A(x)) = \inf\{\mathcal{I}(A(u))\, u \in U_{\mathcal{I}}\} \\
(6) & \mathcal{I}(\exists x A(x)) = \sup\{\mathcal{I}(A(u))\, u \in U_{\mathcal{I}}\}
\end{array}
$$

For a truth value set V, a (possibly infinite) set Γ of formulas (1-)entails a formula A if the interpretation $\mathcal{I}$ on V of A is 1 in case the interpretations of all formulas in Γ are 1, i.e., $\Gamma \Vdash_V A \Longleftrightarrow (\forall \mathcal{I}, \forall B \in \Gamma : \mathcal{I}(B) = 1) \rightarrow \mathcal{I}(A) = 1$.

As a generalization of classical satisfiability, we introduce the following concepts: The formula in Gödel logic is *valid* if the formula evaluates to 1 under every interpretation. The formula in Gödel logic is *1-satisfiable* if there exists at least one interpretation that assigns 1 to the formula.

In Gödel logics, valid prenex formulas can be sharpened to validity equivalent purely existential formulas by Skolemization.

Lemma (Skolemization): For all prenex formulas $Q\bar{x}A(\bar{x})$ and all Gödel logics G

$$\Gamma \Vdash_G Q\bar{x}A(\bar{x}) \Longleftrightarrow \Gamma \Vdash_G (Q\bar{x}A(\bar{x}))^S$$

where $Q\bar{x}$ is a quantifier prefix and $A(\bar{x})$ is a quantifier-free formula.

Proof. It is sufficient to prove with A arbitrary and f a new function:

$$\Gamma \Vdash_G \exists \overline{x} \forall y A(\overline{x}, y) \Leftrightarrow \Gamma \Vdash_G \exists \overline{x} A(\overline{x}, f(\overline{x})).$$

It follows then from induction. ($\Rightarrow$) The direction from left to right is obvious. ($\Leftarrow$) For the other direction, if $\nVdash_G \exists \overline{x} \forall y A(\overline{x}, y)$ then for some interpretation $\mathcal{I}$

$$\sup\{d_{\overline{c}} \mid \mathcal{I}(\forall y A(\overline{c}, y)) = d_{\overline{c}}\} \leq d < 1.$$

Using the axiom of choice we can assign a value for every $f(\overline{c})$ such that $\mathcal{I}(A(\overline{c}, f(\overline{c}))$ is in between $d_{\overline{c}}$ and $d_{\overline{c}} + \frac{1-d}{2}$. As a consequence

$$\sup\{d_{\overline{c}} + \frac{1-d}{2} \mid \mathcal{I}(A(\overline{c}, f(\overline{c}))) \leq d_{\overline{c}} + \frac{1-d}{2}\} \leq d + \frac{1-d}{2} < 1$$

and thus $\Gamma \nVdash_G \exists \overline{x} A(\overline{x}, f(\overline{x}))$. □

Theorem: Validity in Berneys-Schönfinkel (BS) class is decidable for all Gödel logics.

Proof. By Skolemization, validity reduces to checking purely existential formulas, allowing effective decidability. □

Corollary: 1) Herbrand's theorem holds for existential formulas with constants in all Gödel logics. 2) Infinitely-valued Gödel logics coincide regarding validity within the Bernays–Schönfinkel class.

Note that 1) is not trivial as prenex formulas and consequently ∃-formulas for countable Gödel logics are not r.e.[4].

Proposition: Prenex formulas in Gödel logics admit 1-satisfiability iff they are classical saitisfiable.

Theorem: 1-satisfiability in Berneys-Schönfinkel class is decidable for all Gödel logics.

Proof. Direct consequence from classical logic's satisfiability being decidable, as classical and Gödel logic 1-satisfiability coincide in this class. □

All Gödel logics agree on the Bernays–Schönfinkel class regarding 1-satisfiability, whereas only infinitely-valued Gödel logics coincide regarding validity. The Bernays–Schönfinkel fragment of infinitely-valued Gödel logic is precisely the intersection of corresponding fragments of finitely-valued Gödel logics for both satisfiability and validity.

Acknowledgment

The research reported in the paper is a joint work with Mariami Gamsakhurdia and is supported by FWF grant P 36571.

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Normalization in the $\lambda\mu\mu'$-calculus

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Keywords:
Parigot's $\lambda\mu$-calculus, weak and strong normalization, realizability semantics

It was observed in the 1990s, through the works of Murthy [12] and Griffin [9], that the Curry-Howard correspondence can be extended to classical logic. Since then, numerous calculi have been proposed to represent natural deduction or Gentzen-style derivations in classical logic [1, 7, 11, 14, 16, 17]. Among these, we focus on Parigot's $\lambda\mu$-calculus [14] due to its simplicity and its close resemblance to the λ-calculus [2]. Parigot's system was dedicated to establish a connection between logical calculi and natural deduction-style proofs in second-order predicate logic.

The $\lambda\mu$-calculus can be viewed as an extension of the λ-calculus, incorporating classical variables (so-called μ-variables) in the calculus along with their corresponding term formation and reduction rules. More precisely, our investigation focuses on de Groote's version of the $\lambda\mu$-calculus [10], which relaxes Parigot's original formulation by allowing μ-abstractions to appear at arbitrary positions within a term. This removes the restriction that a μ-abstraction must immediately be followed by a μ-application.

In addition to the well-known β-reduction from the λ-calculus, the $\lambda\mu$-calculus introduces several rules specific to classical logic. Namely, the μ-reduction is concerned with the μ-redexes representing reasoning by reductio ad absurdum, whereas its symmetric counterpart, the μ'-rule, is introduced for the μ'-redexes. The μ'-rule is designed to facilitate the uniqueness of representation of data-particularly in the case of Church numerals-within the classical logic setting [14, 13]. It also contains a number of simplification rules: the ρ- and θ-rules, originally introduced by Parigot [15], and the ε-rule, introduced by de Groote [10].

Parigot proved that the $\lambda\mu$-calculus is strongly normalizing [15]. Subsequently, it was shown-using methods formalizable within first-order arithmetic-that strong normalization is also preserved in the untyped $\mu\mu'$-calculus, and that a similar result holds for the simply typed calculus based on the $\lambda\mu\mu'$-rule

[4, 8]. However, the introduction of additional rules has a profound impact on the proof-theoretic properties of the system. In particular, the inclusion of the ρ- or ε-rules leads to the failure of strong normalization [3]. As demonstrated in [5], considering the $\beta\mu\mu'\rho\theta\varepsilon$-reduction rule, the weak normalization can be recovered in the simply typed calculus.

In this talk, we present some existing results in this direction and explore various possible methods for obtaining normalization properties, based on [5, 6].

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Suppes–style probabilistic natural deduction

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Keywords:
inference rules, probability logic, natural deductions, soundness, completeness.

In this paper, we continue our research into the syntactic and semantic properties of logical connectives and probability (v. [1], [2], [3], [4], [8]). Our talk presents a combination of the proof–theoretic ideas of Gentzen and Prawitz [12], Carnap's and Popper's probabilistic concept of a sentence (v. [5], [11]), and Suppes' ε–approach to probabilistic form of inference rules (v. [13], [14]).

Advantage of Suppes' approach to probability of sentence is that arithmetic concerning the probabilities is quite natural and enables to obtain very simple and elegant inference rules. Suppes starts with the probability of a proposition belonging to the interval $[1-\varepsilon, 1]$, for a fixed small positive real ε, and proceeds with inferring new propositions which probabilities belong to the intervals of the form $[1-n\varepsilon, 1]$, for some natural number n.

We consider semantics, through soundness and completeness, as a necessary justification and tool for better understanding of the syntax. On the other side, the syntax presents a bridge from the theory to its application. In this case, each well founded syntactic rule represents a reliable scheme for an algorithmic step in a programm for calculating appropriate probability of a given sentence. Consequently, the syntactic part of our logic **NKprob**(ε) can be treated as a framework for a programm producing an output A^m for any input $(A_1^{m_1}, \ldots, A_n^{m_n})$ (see also examples given in [4]). The denotation **NKprob**(ε) come from **NK** — for Gentzen's original natural deduction system for classical propositional logic, **prob** — for probability, and ε — for Suppes' ε–approach.

The basic form of formulae of our system is A^m, with meaning that 'the probability of A belongs to $[1-m\varepsilon, 1]$'. An auxiliary form of formulae is A^{-m}, with meaning that 'the probability of A belongs to $[0, m\varepsilon]$, for $m \in \mathbf{N}$'. The

set of propositional formulae is build up inductively over a denumerable set of propositional letters, two constants, $\top$ and $\bot$, and the usual set of propositional connectives $\{\neg, \wedge, \vee, \rightarrow\}$.

The system **NKprob**(ε) consists of introduction ($I\star$) and elimination ($E\star$) rules, for each propositional connective $\star \in \{\neg, \wedge, \vee, \rightarrow\}$, and some specific rules concerning probability properties. For instance, the nature of *implication* is covered by the rules:

$$\frac{(\neg A)^m \quad B^n}{(A \rightarrow B)^{\min\{m,n\}}}(I \rightarrow) \qquad \frac{A^m \quad (A \rightarrow B)^n}{B^{m+n}}(E \rightarrow)$$

Our perception of models for **NKprob**(ε) will be founded on basic properties of sentence probability as considered by Carnap, Popper, Leblance, van Fraassen, Hailperin (v. [5], [6], [7], [9], [10], [11], [14]) and, finally, by Suppes [13]:

(i) $p(A) = 1$, for each classical tautology A;

(ii) if $A \leftrightarrow B$ is a classical tautology, then $p(A) = p(B)$, and

(iii) if $p(A \wedge B) = 0$, then $p(A \vee B) = p(A) + p(B)$.

We prove that our system **NKprob**(ε) is sound and complete with respect to the presented Carnap–Popper type of semantics.

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Interpretable vs. Learnable Centrality: Combining SLI and Neural Networks

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Keywords:
complex networks; undirected graphs; directed graphs; weighted graphs; centrality measures; node importance; key nodes; neural networks.

With the increasing size and structural richness of complex networks, the identification of key nodes in a specific context becomes increasingly important in order to understand, optimise and secure modelled systems. In our previous works, we proposed the Semi-Local Integration Measure (SLI) and its directed variant Directed Semi-Local Integration Measure (DSLI), centrality measures for evaluating node integration [1, 3]. These measures have already been successfully used in Natural Language Processing and have proven to be effective in capturing nuanced local structural properties that are often not recognised by traditional existing centrality measures [2].

In this new research phase, we are exploring a data-driven, AI-assisted approach to centrality by embedding SLI/DSLI in neural network architectures. The goal is to develop adaptive centrality models that are able to generalise node importance patterns across different graph topologies and tasks. By using SLI/DSLI as a supervision component, we aim to combine mathematically interpretable centrality formulations with the flexibility and adaptability of modern machine learning algorithms. While SLI/DSLI measures are based on precise and theory-driven definitions that reflect node integration, modern algorithms provide the ability to learn task-specific patterns and adapt to the structure of the specific complex network.

Our hybrid approach improves the transparency and interpretability of AI models by integrating data-driven patterns that attempt to bridge the gap between traditional explainability and modern learning techniques. We expect a context-aware evaluation of node roles and a way to integrate learned centrality into various applications, such as influence detection in social networks, anomaly detection in communication infrastructures, structural analysis of semantic graphs in NLP, etc. This is a step towards AI systems that are not only powerful, but also interpretable, explainable and adaptable to different tasks and domains.

Acknowledgment

This work has been supported by the University of Rijeka under the project *Mathematical Modelling in NLP* (uniri-iskusni-prirod-23-150) and Croatian Science Foundation under the project Formal Systems and Modelling (HRZZ-IP-2024-05-3882).

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Computational Complexity of Some Time-Bounded Verification Problems

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Keywords: Resilience, Realizability, Survivability, Computational Complexity, Multiset Rewriting, Maude

Resilient systems are systems able to adapt to unexpected changes or adversarial disruptions. Here we study the computational complexity of the formalization of the time-bounded resilience problem for the class of η-simple progressing planning scenarios, where, intuitively, it is simple to check that a system configuration is critical, and only a bounded number of rules can be applied in a single time step.

We show that, in the time-bounded model with n (adversarially chosen) disruptions, the corresponding time-bounded resilience problem for this class of systems is complete for the $\Sigma^{\mathsf{P}}_{2n+1}$ class of the polynomial hierarchy, PH.

We also consider the computational complexity of several related problems, such as time-bounded realizability (Σ^P_2-complete) and time-bounded survivability (DP-complete).

We also present two illustrative examples: the travel-planning example and the production supply chain example. If time permits, we will discuss automated experiments for time-bounded verification using the rewriting logic tool Maude.

Acknowledgment

Ban Kirigin is supported by Croatian Science Foundation under the project HRZZ-IP-2024-05-3882.

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How to extend the set-theoretic vocabulary in a type-safe way?

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Keywords:
set theory, conservative extension, type safety, expressive power, iterative language enrichment

In set theory, it is customary to introduce new nonlogical (relation, function, and constant) symbols all the time. We can't even state all the axioms of ZF in the same language, since (for example) for the statement of the Axiom of infinity, we need a constant symbol for the empty set and a function symbol for the set union.

This process of iteratively expanding the language and using it to define new kinds of formulas (which then in turn define new constructs of the language) is well-known, and utilized all the time in developing the foundations of mathematics. However, it is not type-safe: we can, at least in theory, apply any function symbol to any terms, no matter what kind of object it denotes.

New Foundations was introduced by Quine in 1937 as a form of a simplified version of Russell-Whitehead type theory. It can also serve as a foundation for mathematics, but done in a type-safe way. Its types are simply natural numbers (or integers), but it turns out to be quite enough to eliminate the usual paradoxes (such as Russell's).

In NF/NFU, we would like to do the same process of expanding the language, but there is an additional complexity regarding stratifiability. While it is easy to say when a formula in the basic language is stratified, it is harder to do so when the formula contains other nonlogical symbols (besides $=$, $\in$, and *set*).

We present one approach, based on *signatures* (tuples of type differences), which gives us a clean way to generalize stratification conditions and to prove that (up to some technical details regarding constant terms) stratifiability remains the same, no matter how many new symbols we add to the language. We will also present a working program in Python designed for expressing formulas in such an extensible language.

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Computability of spaces with cylindrical ends

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Keywords:
Computable metric space, semicomputable set, computable set, noncompact space.

A set S in a computable metric space is *semicomputable* if

(i) its intersection with any closed ball is compact; and

(ii) it is possible to effectively enumerate all finite unions of basic open balls which cover $S \cap \overline{B}_i$, uniformly over rational closed balls $\overline{B}_i$.

If, in addition, there is an effective procedure to enumerate all basic open balls that intersect S, then S is said to be *computable*.

Topological properties can play a significant role in determining whether a semicomputable set is computable. Specifically, if every semicomputable set homeomorphic to a space A is computable, we say that A has *computable type*. More generally, a topological pair (A, B), consisting of a space A and its subspace B, is said to have computable type if, whenever $f : A \to X$ is an embedding into a computable metric space such that both $f(A)$ and $f(B)$ are semicomputable, then $f(A)$ is computable.

The study of computable type has traditionally focused on compact spaces, particularly manifolds and simplicial complexes [3, 1]. However, more general approaches have led to meaningful results in the noncompact setting as well. For example, it was shown in [2] that any semicomputable 1-manifold in a computable metric space is computable. This result was later extended to generalized graphs in [5]. Furthermore, in [4], it was proved that a semicomputable manifold M of arbitrary dimension is computable if there exists a relatively compact open subset $U \subseteq M$ such that $M \setminus U$ is homeomorphic to $\mathbb{R}^n \setminus \mathbb{B}^n$.

In this talk, we begin with a survey of the techniques used to obtain the aforementioned results for noncompact spaces. We then turn our attention to the notion of *pseudocompactification*, introduced in [4], which enables us to apply established results from the compact setting. Finally, we examine *neighborhoods*

of infinity (that is, complements of compact subsets) in noncompact manifolds, and show how their topological properties affect computable type. We prove the following:

Theorem 1. *Suppose S is a semicomputable set in a computable metric space which contains a closed neighborhood of infinity homeomorphic to $Q \times [0,1\rangle$ for some compact space Q. If the cone $\mathsf{C}(Q)$ has local computable type, then S is computable at infinity.*

As a direct consequence, we get that the infinite cylinder $\mathbb{S}^1 \times \mathbb{R}$ (and $\mathbb{S}^n \times \mathbb{R}$ in general) has computable type.

This talk is based on joint work with Zvonko Iljazović.

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Incorrect Proofs and Epsilon Calculus

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Keywords:
Incorrect Proof, Epsilon calculus, Herbrand Disjunction.

Many significant mathematical discoveries initially came accompanied by incorrect or incomplete proofs, famously exemplified by Euler's first attempt at solving the Basel problem [5], [3]. Such historical instances prompt two central questions in mathematical proof theory:

1. If a proof is incomplete but intuitively reasonable, what is the minimal additional information required to complete and validate it?

2. If a proof is reasonable yet incorrect, how can we systematically identify a suitably weakened statement that can be correctly established through a closely related argument?

These questions are inherently dual: an incomplete proof of a suitably weakened statement, when completed, yields a valid proof of an implication towards the originally intended result. This talk delves into formal logical frameworks to precisely determine minimal or weakest preconditions necessary to rectify proofs. Such weakest preconditions are understood as preconditions implied by all other possible preconditions, representing the simplest means of ensuring the correctness of a given argument.

In classical propositional logic, identifying weakest preconditions is straightforwardly achievable through truth tables; As the non-validity of a formula can be identified with its incorrect proof: the minimal information to make, e.g., $A \wedge B \to A \wedge C$ valid is $\neg(A \wedge B \wedge \neg C)$ excluding the only line of the truth table falsifying the formula. This means

$$\neg(A \wedge B \wedge \neg C) \to (A \wedge B \to A \wedge C)$$

is a tautology.

However, first-order logic introduces significant complexities, including the undecidability of determining weakest preconditions, as "$\bot$ is the minimal information to make A valid" is equivalent to "A is valid". Furthermore, such a weakest information might not exist. For example,

$$\forall x(A(p(x)) \to A(x)) \to A(0)$$

has, using Herbrand's theorem, the validating premises $A(0), A(p(0)), A(p(p(0)))$, ... and $A(p^{n+1}(0))$ is weaker then $A(p^n(0))$. Consequently, analysis in first-order logic requires examining specific incorrect proofs, often formulated within a sequent calculus framework.

To address these complexities, we employ Hilbert's epsilon calculus [2],[4],[1], a formalism based on the replacement of $\exists x A(x)$ by $A(\varepsilon_x A(x))$ and $\forall x A(x)$ by $A(\varepsilon_x \neg A(x))$, enabling first-order proofs to be analyzed in a manner similar to propositional logic. By systematically translating proofs into epsilon calculus, we can leverage methods akin to propositional analysis to ascertain weakest preconditions and correction strategies.

Example: The standard translation of $\exists x A(x) \wedge \forall x B(x)$ is $A(\varepsilon_x A(x)) \wedge B(\varepsilon_x \neg B(x))$.

In this talk we will systematically introduce and develop proof-theoretic tools based on ε-calculus for analyzing and correcting flawed mathematical proofs. Furthermore, we will discuss recent theoretical results demonstrating the robustness, or false-tolerance, of epsilon calculus, ensuring that even in cases of minimal errors, useful computational information can still be extracted from incorrect proofs.

As an example, we provide the following theorem.

Theorem: The algorithm of the extended first ε-theorem is false-tolerant: if there is at most only one interpretation that falsifies the proof (i.e., one line of the minimal truth table) then the same holds for the Herbrand disjunction obtained after elimination of critical formulas.

Acknowledgment

The research reported in the paper is a joint work with Ao.Univ.Prof. Dr. Matthias Baaz and is partly supported by FWF grant P 36571.

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Logical foundations for correct communication of distributed machine learning

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Keywords:
Communicating Sequential Processes, Multiparty Session Types, Federated Learning, Verification.

Distributed machine learning setting where clients collaboratively train and communicate a model while keeping the training data decentralised and local is called Federated Learning (FL). FL can be centralised or decentralised, where the collaborative model training is under the coordination of a central server, or a peer-to-peer network, respectively. The correct orchestration of FL systems is challenging. In FL, communication protocols follow key patterns that must be expressed for proper modelling and verification.

In this talk, we present two approaches to formal verification of the correctness of generic FL algorithms:

- untyped calculus and model checking, presented in [1],
- typed calculus and correctness-by-construction, presented in [2].

Untyped calculus and model checking, [1] FL algorithms are modelled in the untyped calculus Communication Sequential Processes (CSP). Correctness of FL algorithms and required properties such as deadlock-freedom and termination are verified by applying the model checker Process Analysis Toolkit (PAT).

Typed calculus and correctness-by-construction, [2] FL algorithms are modelled in an extension of Multiparty Session Types system (MPSTs), which is specially developed for this purpose. As a novelty MPSTs supports input/output operations directed towards multiple participants at the same time.

*Partially funded by the European Union, project TaRDIS no. 101093006.

The MPSTs is proven to enjoy safety, deadlock-freedom, liveness, and session fidelity properties. FL algorithms are modelled in MPSTs, which paves the way for more scalable and efficient techniques for verification and analysis of distributed machine learning algorithms based on correctness-by-construction.

This talk is based on joint work with Miodrag Djukić, Ivan Kaštelan, Miroslav Popović, Marko Popović, Ivan Prokić, Simona Prokić, Alceste Scalas and Nobuko Yoshida.

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Theory and Applications of Orthologic

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I will present the theory of orthologic, and its applications to predictable verification and theorem proving.

1 Orthologic

Specialized, reliable, and efficient building blocks are indispensable in scaling automated reasoning software. Program verifiers, SMT solvers, proof assistants, and automated theorem provers use them to tackle the various theories and subproblems that comprise a logical statement.

One such fragment of particular interest is propositional logic. Despite progress in SAT solvers, solving satisfiability or validity of propositional formulas remains a major challenge to scalability of decision procedures. An alternative approach to heuristics is orthologic-based reasoning. Orthologic is a non-distributive generalization of classical propositional logic which admits $\mathcal{O}(n^2)$ validity checking and normalization algorithms [1, 2, 7]. Orthologic offers a trade-off: it sacrifices completeness (with respect to classical semantics) in exchange for guaranteed efficiency and predictability. The algebra underlying orthologic is an *ortholattice*, whose laws are presented in the following table:

V1:	$x \vee y = y \vee x$	V1':	$x \wedge y = y \wedge x$
V2:	$x \vee (y \vee z) = (x \vee y) \vee z$	V2':	$x \wedge (y \wedge z) = (x \wedge y) \wedge z$
V3:	$x \vee x = x$	V3':	$x \wedge x = x$
V4:	$x \vee 1 = 1$	V4':	$x \wedge 0 = 0$
V5:	$x \vee 0 = x$	V5':	$x \wedge 1 = x$
V6:	$\neg\neg x = x$		
V7:	$x \vee \neg x = 1$	V7':	$x \wedge \neg x = 0$
V8:	$\neg(x \vee y) = \neg x \wedge \neg y$	V8':	$\neg(x \wedge y) = \neg x \vee \neg y$
V9:	$x \vee (x \wedge y) = x$	V9':	$x \wedge (x \vee y) = x$

Key properties of orthologic are as follows:

- Orthologic admits a quadratic-time normalization algorithm, mapping equivalent formulas to one formula of minimum size [2].

- Orthologic admits a proof system, supporting arbitrary axioms, with cut elimination and subformula property. Proof search has complexity $\mathcal{O}(n^2m)$, where n is the size of the formulas and m the number of axioms [7].
- Normalization and proof search still hold with the same complexity in th presence of uninterpreted monotonic function symbols (not yet published).
- Orthologic can be extended with predicates, and is classicaly complete for classes of formulas such as Horn clauses and Datalog programs [7].
- Orthologic admits efficiently computable interpolants [6].

2 Results

Orthologic admits a natural sequent-based proof system, which consists in restricting the classical sequent calculus so that at any given time a sequent never contains more than two (different) formulas. This system admits cut elimination, which we showed generalizes accordingly when a set of non-logical axioms is allowed as the leaves of the proof. Using this property, orthologic with arbitrary axioms is decidable in time $\mathcal{O}(n^2m)$. We have verified this decision procedure in the Rocq proof assistant [8].

Orthologic (with axioms) is classically complete with respect to Horn clauses and other relevant classes of formulas. Orthologic can also be extended to support first-order variables and predicate symbols, similarly as in classical effectively propositional logic (EPR) and Datalog. In fact, orthologic is complete for Datalog programs and can be used as a proper extension of Datalog. Orthologic also admits the *interpolation property*, i.e. for two formulas ϕ, ψ, there exists γ containing only the shared variables of ϕ and ψ, and such that $\phi \leq \gamma \leq \psi$. Moreover, unlike in classical logic, such interpolants can always be computed efficiently. This makes orthologic promising for interpolation model checking.

In [2], we presented a quadratic-time normalization procedure for orthologic. We implemented it in the Stainless program verifier[9], where it serves two purposes: first, it reduces the size of verification conditions, leading to faster solving. Sometimes, formulas are even found to be true without needing to be sent to the SMT solver. Second, caching normalized formulas increases the cache hit ratio of verification conditions.

Another application of orthologic is as the basis of a type system with subtyping. union and disjunction types have been adopted by popular languages such as Scala and Typescript, but deciding the subtyping relation remains a practical challenge. Type checkers are often incomplete, cannot effectively reason with assumptions, and sometimes rely on unsound simplifications. Considering types as members of an ortholattice allows to decide reliably and efficiently subtyping relations. Moreover, orthologic normalization and proof search procedure extend in the presence of monotonic and antimonotonic function symbols, corresponding to covariant and contravariant type constructors. Supporting assumptions then allow us to represent features of type systems such as record

types, f-bounded and constrained polymorphism, recursive types, inheritance, complement types, and more.

3 Orthologic in a Proof Assistant: Lisa

The initial motivation behind orthologic was the following: In a formal proof system, many intuitively obvious transformations (for example, swapping the order of disjuncts) must be explicitly proven. This makes proofs longer, less efficient, and frustrating to write. Orthologic is the answer to the question "Can we find an efficient algorithm with clear completeness guarantees that solves automatically many of these obvious transformations?".

The system in question is *Lisa* [5], a proof assistant made from scratch and based of first-order logic and set theory. This is most generally accepted among mathematicians as the foundation of mathematics, offers a lot of power and expressibility, and has been stable and without hints of inconsistencies for a hundred years. Hence was created Lisa, with a focus on 6 principles for proof systems: trust, efficiency, predictability, usability, interoperability, and programmability. For example, the development of orthologic was motivated by the need to find the best compromise between efficiency, predictability, and usability. In practice, orthologic simplification allows for simpler and shorter proofs and tactics.

Programmability is a key focus of Lisa compared to most older proof assistants. Lisa has a unified implementation, proof writing, and tactic language (Scala) and comes with a domain-specific language (DSL) to write proofs in a natural way, with a "*have* statement *by* tactic" syntax. Such proof scripts using the DSL are fully executable code, computing a kernel proof. Moreover, the proof DSL can be used when writing tactics, and can freely be mixed with regular programming constructs and with the entire standard libraries of Scala and Java. Using this DSL, a variety of proof-producing decision procedures and formalized theorems have been implemented. One such tactic, called `tautology`, decides propositional tautologies by alternating between computing orthologic normalization and branching on a literal. In practice, this yields a significantly better algorithm than doing a simplification with constant folding

In [4], we describe how to mechanize HOL-style proofs into first-order set theory, supporting functions and sets as simple types with top-level polymorphism, in the style of Hindley-Milner. The technical difficulties in the representation of λ-terms in strict first-order logic lead to the development of λFOL, a syntactic extension of first-order logic with some higher-order expressions. λFOL is closer to vernacular mathematics, allowing expressions such as set comprehensions ($\{x \in \mathbb{N} \mid \exists y \in \mathbb{N}.x = y + y\}$), but otherwise fundamentally stays first-order. λFOL, alongside sequent calculus and orthologic, now make the foundation of Lisa. Extending the type system to support union, negation and interesection types with orthologic semantics, as described above, is ongoing work.

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Cut-elimination for non-wellfounded sequent calculi for IL

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Keywords:
Cut elimination, Interpretability logic, Sequent calculi.

Interpretability logic extends provability logic, which is a modal treatment of Gödel's provability predicate. Its language has an additional binary modality, which corresponds to the notion of relative interpretability between first-order arithmetical theories. Although many papers on the semantics of interpretability logics have appeared in the recent years, sequent systems for interpretability logics were not much investigated. One appeared in [3], but it requires a complicated proof of the cut elimination.

In recent decades a non-wellfounded proof theory has gained prominence. It results from allowing proofs to have infinite height. In this talk, we will present the wellfounded sequent system GIL and non-wellfounded sequent calculus $\mathsf{G}^{\infty}\mathsf{IL}$ for the interpretability logic IL that we developed in [1]. Finally, we will show how to prove cut elimination for these systems in much simpler way. In order to do that we will use the method presented in [2].

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Computable subcontinua of circularly chainable continua

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Keywords:
computable metric space, circularly chainable continuum, chainable continuum, semicomputable set, computable set.

In this talk we explore, in computable metric spaces, circularly chainable continua which are not chainable. Given such a continuum K, if we endow it with semicomputability, its computability follows. Conditions under which semicomputability implies computability, typically topological, are extensively studied in the literature [Ilj]. When a semicomputable set K is not computable, it is natural to explore approximate approaches; under certain conditions such a set can be approximated by computable subsets [IP]. Two general questions motivate this work. Given a (semi)computable continuum K,

1. Under which conditions does K contain computable subcontinua?
2. What can be said about the existence of computable points inside designated subsets of K?

The main result establishes that, given two points on a semicomputable, circularly chainable, but non-chainable continuum K, one can approximate them by computable points such that there exists a computable subcontinuum connecting these approximations. As a consequence, given disjoint computably enumerable open sets U and V intersected by K, the intersection of K with the complement of their union necessarily contains a computable point, provided that this intersection is totally disconnected.

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Computable categoricity and subspaces of Euclidean space

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We examine effective separating sequences on a metric space and, in particular, conditions under which on a metric space every two such sequences are equivalent up to an isometry. Such a metric space is called computably categorical. We prove that an effectively compact metric space (X, d) is computably categorical if the space of all isometries of (X, d) has computable type. Using this, we prove the following result.

Theorem 1 *Every effectively compact subspace of Euclidean space is computably categorical.*

Acknowledgment

The research reported in the paper is partly supported by some fine projects.

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Computability of common fixed points of isometries

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Keywords:
Computability, Isometries, Common fixed points, Convexity.

Let (X, d) be a metric space and $S \subseteq X$. An isometry of S is a function $f : S \to S$ such that for all $x, y \in S$,

$$d(f(x), f(y)) = d(x, y).$$

If S is compact it is well known that every isometry of S is bijective. Define,

$$F(S) = \{x \in S \mid f(x) = x, \text{ for all isometries } f : S \to S\},$$

the set of common fixed points of all isometries of S. In this work, we focus on compact subsets of $\mathbb{R}^n$ equipped with standard Euclidean metric, and study the computability of the set $F(S)$. It follows from theorem of Brodskii and Milman ([3]) that if $S \subseteq \mathbb{R}^n$ is convex and compact, then $F(S)$ is nonempty. Our main result establishes that if S is a computable and convex subset of $\mathbb{R}^n$, then the set $F(S)$ is computable. We also provide an example of a non-convex computable set S such that $F(S)$ is semicomputable, but contains no computable point.

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On the Proof Complexity of Several Combinatorial Principles: the role of Kernelization

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Keywords:
Propositional proof complexity, Frege proofs, kernelization.

1 Introduction

In this presentation, based on papers [7, 1, 8] (written in colaboration with several coauthors from West University of Timişoara, University of California San Diego and Universitat Politécnica de Catalunya), as well as on more recent, unpublished, research, we discuss the use of combinatorial statements as a source of interesting candidate propositional formulas in *proof complexity*. An important open problem in this area is to find "natural" candidates for separating the Frege and extended Frege propositional proof systems (see [9] for an in-depth presentation). We had originally conjectured that the so-called Kneser-Lovász theorem (see e.g. [10]) is such a candidate. But in [1] we disproved this conjecture, proving the existence of quasi-polynomial size Frege proofs for the Kneser-Lovász theorem. The strategy employed in this proof was abstracted and generalized in [8], allowing us to prove the existence of polynomial (or quasi-polynomial, in some cases) Frege and extended Frege proofs for a variety of propositional encodings of several combinatorial principles, such as

- Schrijver's theorem [11], a generalization of the Kneser-Lovász theorem[1]
- vertex coloring, dual graph coloring, edge clique cover, hitting set.
- the Arrow and Gibbard-Satterthwaite theorems from the theory of social choice.

In the present talk:

[1] For some recent applications of our proof strategy for this result see [4]

- We aim to explain some of the technical details of our results in [8], specifically how the concepts of *data reduction/kernelization* from parameterized complexity [2]

- Time permitting, we will present subsequent research directions/work:

    1. First, the application of a different technique from the theory of parameterized complexity, that of *iterative compression* [3].
    2. Second, the applications of our techniques to proof systems other than Frege and extended Frege. Our main target is the class of proof systems based on clause redundancy recently introduced in the SAT solving community [5, 6], specifically the proof system SPR^{-} [12]. These are proof systems for which the logical equivalence of formulas generated through the proof is **not** guaranteed. Instead, these formulas are only *equisatisfiable* with original formulas. Such proof systems have important practical advantages, and the issue of guaranteeing efficient proofs in them is a practically important one.

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Fine-Tuning LLMs for Croatian Text Retrieval

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Keywords:
Text retrieval, Embeddings, Large language models, Fine-tuning.

We investigate the possibility of adapting large language models (LLMs) to generate high-quality text embeddings in Croatian, a low-resource language. Text embeddings are vector representations of text used in retrieval tasks, where the goal is to find semantically similar documents based on vector distances rather than relying on exact keyword matching. Therefore, generating quality text embeddings is a key component of non-keyword-based text retrieval systems.

In our study, we fine-tune several LLMs such as Mistral, LLaMA, and Gemma, as well as embedding models, including BGE—using parameter-efficient fine-tuning. We identify BGE-HR, our adapted version of BGE, as the best-performing model. Rather than focusing solely on introducing a new model, we conduct a systematic analysis of the factors that influence its successful adaptation to low-resource languages. An important aspect of this analysis involves evaluating the ability of base LLMs to generate coherent text in Croatian, including how effectively they segment text into smaller units (tokens) for subsequent processing.

To further assess model generalizability, we introduce a new evaluation dataset derived from commonly asked questions found on the Croatian web. This enables us to benchmark performance on realistic text retrieval tasks that differ from the dataset used for fine-tuning. Our findings suggest that choosing a stronger base model is generally more effective than investing substantial effort in fine-tuning a weaker one. We also find that some smaller, publicly available multilingual embedding models, generalize well on the new dataset, offering a practical solution when computational resources for fine-tuning are limited.

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Amalgamation in classes of involutive commutative residuated lattices

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1 Introduction

Amalgamation is explored in this talk within classes of involutive commutative residuated lattices that are non-divisible, non-integral, and non-idempotent. Several classes of algebras significant to us are designated by a distinctive notation:

$\mathfrak{A}^{\mathfrak{c}}$	the class of abelian o-groups
$\mathfrak{I}$	the class of involutive FL_e-algebras
$\mathfrak{S}$	the class of odd or even idempotent-symmetric involutive FL_e-algebras

Adjunct to $\mathfrak{I}$,

- the superscript $\mathfrak{c}$ means restriction to totally-ordered algebras,
- the superscript $\mathfrak{sl}$ means restriction to semilinear algebras,
- the subscript $\mathfrak{o}$ means restriction to odd algebras,
- the subscript $\mathfrak{e}$ means restriction to even algebras,
- the subscript $\mathfrak{e}_{\mathfrak{i}}$ means restriction to even algebras having an idempotent falsum constant,
- the subscript $\mathfrak{e}_{\mathfrak{n}}$ means restriction to even algebras having a non-idempotent falsum constant,

When multiple letters appear in the subscript, they denote the union of the corresponding classes. For instance $\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{o}\mathfrak{e}_{\mathfrak{i}}}$ refers to the class of idempotent-symmetric involutive FL_e-chains which are either odd or even with an idempotent falsum constant.

First we delve into the Amalgamation Property within subclasses of $\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{o}\mathfrak{e}}$. We show that several subclasses of these structures fail to satisfy the Amalgamation Property (Theorem 2.1), including the classes of odd and even ones. This failure stems from the same underlying reason as in the case of discrete linearly ordered abelian groups with positive normal homomorphisms [3]. Conversely, it is proven that three subclasses of them exclusively comprising algebras that

are idempotent-symmetric possess the Amalgamation Property (Theorem 2.2), albeit fail the Strong Amalgamation Property (Theorem 2.3). The failure of the Strong Amalgamation Property in these subclasses can be attributed to the same underlying reason observed in the class of linearly ordered abelian groups with positive homomorphisms [1].

Then we shift our focus from these classes of chains to the semilinear varieties of FL_e-algebras that they generate. Our goal is to transfer the Amalgamation Property, or its failure, from the specific classes of chains to the generated varieties. We conclude that every variety of semilinear involutive commutative (pointed) residuated lattices that includes the variety of odd semilinear commutative residuated lattices fails the Amalgamation Property (Theorem 3.1). This result strengthens a recent proof by W. Fussner and S. Santschi, which established that the variety of semilinear involutive commutative residuated lattices lacks the Amalgamation Property [2, Theorem 5.2]. Furthermore, we demonstrate that the varieties of idempotent-symmetric, semilinear, odd involutive residuated lattices, as well as idempotent-symmetric, semilinear, odd or even involutive residuated lattices, exhibit the Transferable Injections Property (Theorem 3.2), a strengthening of the Amalgamation Property.

2 Amalgamation in classes of $\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{oe}}$

Theorem 2.1. *The classes $\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{e}}$, $\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{e}_{\mathfrak{i}}}$, $\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{e}_{\mathfrak{n}}}$, along with every class of involutive FL_e-chains which contains $\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{o}}$, fail the Amalgamation Property.*

Theorem 2.2. *The classes $\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{o}}$, $\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{e}}$, and $\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{oe}}$ each satisfy the Amalgamation Property.*

Theorem 2.3. *The classes $\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{o}}$, $\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{e}}$, and $\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{oe}}$ do not satisfy the Strong Amalgamation Property.*

3 Amalgamation in the generated semilinear varieties

Theorem 3.1. *Every variety of semilinear involutive commutative (pointed) residuated lattices that includes the variety of odd semilinear commutative residuated lattices fails the Amalgamation Property.*

Theorem 3.2. *The varieties $\mathfrak{S}^{\mathfrak{sl}}_{\mathfrak{o}}$ and $V(\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{e}})$ have the Transferable Injections Property.*

4 Techniques

The core principle of our approach relies on leveraging the intrinsic layer group decomposition of the algebras in $\mathfrak{I}^{\mathfrak{c}}_{\mathfrak{oe}}$ [4] and an associated categorical equivalence [5]. This strategic direct system decomposition facilitates the independent execution of amalgamation within each distinct layer. Subsequently, these layer-wise amalgams are leveraged to construct the overall amalgam of the algebras via the functor detailed in [5] (see Fig. 1).

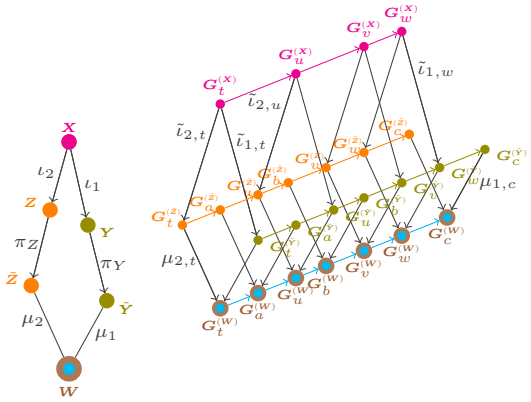

Figure 1: Brief visual illustration of the main constructions: "Layerwise" amalgamation in $\mathfrak{A}^{\mathfrak{c}}$ (right), and the corresponding amalgamation in $\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{o}\mathfrak{c}}$ (left).

As an example, proving Theorem 4.1 was necessary to convert the cyan direct system into the brown one. Additionally, several techniques for embedding direct systems into those over larger index sets were developed to construct the embeddings shown in Fig. 1.

Theorem 4.1. *For any direct system $\langle \boldsymbol{L}_u, \varsigma_{u\to v}\rangle_\kappa$ of torsion-free partially ordered abelian groups over an arbitrary chain κ, there exists a direct system $\langle \widehat{\boldsymbol{G}}_u, \varsigma_{u\to v}\rangle_\kappa$ of abelian o-groups. In this system the abelian group reducts of the $\boldsymbol{L}_u$'s and the transitions remain unchanged, while, for every $u \in \kappa$, the ordering relation of $\widehat{\boldsymbol{G}}_u$ is an extension of the ordering relation of $\boldsymbol{L}_u$.*

Acknowledgment

This research was supported by the NKFI-K-146701 grant.

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Exponentiation and Iteration in the Lambek Calculus and Its Variants

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Keywords:
Lambek calculus, Kleene star, linear logic, algorithmic complexity.

The *Lambek calculus,* originally introduced for mathematical modelling of natural language syntax [1], is now considered as a basic *substructural* logical system. Namely, the sequent-style formulation of the Lambek calculus uses basically the same logical rules as for intuitionistic logic, but lacks the *structural* rules: contraction, weakening, and permutation. The only implicit structural rule is associativity, which is removed in the even more restrictive *non-associative Lambek calculus.* In the absence of contraction, the Lambek calculus enjoys bounded cut-free proof search, and therefore it is algorithmically decidable, and so are many of its extensions with additional connectives. In this talk, we shall discuss two unary operations which, being added to the Lambek calculus, attack its decidability and yield interesting complexity results.

The first operation is the *exponential,* coming from Girard's [2] linear logic. Under the exponential, structural rules are allowed, which (in the associative case) leads to undecidability [3]. A more fine-grained control of structural rules is obtained by considering families of *subexponentials* [4, 5, 6]. Subexponential modalities have some yet underexplored similarity to other ways of structural control in the (non-associative) Lambek calculus, including multi-modal systems [7] and the Lambek calculus with brackets [8].

The second operation we consider is *iteration,* or *Kleene star.* One of its natural axiomatisations is infinitary and yields a Π^0_1-complete logic [9]. In the non-associative case, iteration is replaced by so-called *iterative divisions* [10].

Even more intriguingly, a combination of (sub)exponentials and Kleene star gives rise to extensions of the Lambek calculus whose complexity rises up to Π^1_1-completeness, with a properly hyperarithmetical $\Sigma^0_\omega{}_\omega$-complete fragment in between [11, 12].

In this talk, we shall give a survey on algorithmic and other properties of extensions of the Lambek calculus with (sub)exponentials and Kleene star (mostly in its infinitary form), from well-known old results up to new ones.

Acknowledgment

This talk is partially based on joint work with Eben Blaisdell, Max Kanovich, Glyn Morrill, Vivek Nigam, Elaine Pimentel, Tikhon Pshenitsyn, Andre Scedrov, and Stanislav Speranski, all of whom the author is indebted to. The work of S. K. was performed at Steklov International Mathematical Centre and supported by the Ministry of Science and Higher Education of the Russian Federation (agreement no. 075-15-2025-303).

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Polytopological semantics of interpretability and conservativity logic

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Keywords:
topological semantics, conservativity logic, interpretability logic, Veltman semantics

Interpretability logic **IL**, introduced by Visser [4], and conservativity logic **CL**, introduced by Ignatiev [2], are extensions of provability logic **GL** with binary modal operator $\triangleright$. One of the most well-known semantics for **IL** and **CL** is the Veltman semantics for which **IL** and **CL** are complete, but not strongly complete.

Iwata and Kurahashi [3] introduced topological semantics for **IL** and **CL** that corresponds to lesser known Visser semantics, sometimes also called simplified Veltman semantics. They also proved topological strong completeness of **IL**, **CL** and some of their extensions.

In this talk, we introduce polytopological spaces and topological semantics for **IL** and **CL** based on them and show that it corresponds to Veltman semantics.

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Finite model property and decidability of inquisitive neighborhood logic

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Keywords:
inquisitive neighborhood logic, selection method, filtration method, decidability.

Inquisitive logic is a generalization of classical logic that can express questions. Its language extends the language of classical logic by adding a connective $\mathbin{\mathbb{V}}$, called inquisitive disjunction, which allows us to express formulas that represent questions. Since questions cannot simply be true or false, semantics is defined via a relation between sets of worlds and formulas, rather than between individual worlds and formulas (cf. [3] and [4]).

A natural generalization of basic modal logic is inquisitive modal logic InqML. Its language is based on the modal operator $\boxplus$, called window, which plays a similar role to the modal operator $\Box$ in basic modal logic. Formulas of InqML are interpreted over models in which each world w is associated with a set of sets of worlds $\Sigma(w)$, satisfying a downward closure condition: if $s \in \Sigma(w)$ and $t \subseteq s$, then $t \in \Sigma(w)$.

In [1] the logic InqML was generalized to inquisitive neighborhood logic InqNL (also cf. [2]). The language of InqNL is based on a binary modal operator $\Rrightarrow$, called yields, while the formulas of InqNL are interpreted over models in which a downward closure condition is not required.

In this talk, we first show that inquisitive neighborhood logic has the finite model property. We use two well-known techniques: the selection method and the filtration method (cf. [6, 5] for their application in InqML). As a consequence of the finite model property, we show that the logic InqNL is decidable.

Acknowledgment

This research was supported by the Croatian Science Foundation under project HRZZ-IP-2024-05-3882.

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Symbolic Computation and Verification Methods in Maude

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Maude and its formal tools support ten different symbolic computation features. This talk will focus on one of them, namely, narrowing-based model checking based on symbolic states described by constrained patterns, as well as its combination with inductive theorem proving in a novel style of deductive model checking. This will illustrate how Maude's symbolic features can be used to model check modal logic properties of infinite-state systems specificed as rewrite theories in Maude.

Two Entensions of Lambek Calculus

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The 1985 Tucson conference Categorial Grammars and Natural Language Structures was a key event in the revival of Lambek-style categorial type logics. Forty years later, a number of variations on the original Syntactic Calculus have been introduced, motivated by logical and/or linguistic considerations. For the application to natural language grammars, these exended calculi generally rely on a combination with Montague's view on the syntax-semantics interface as a structure-preserving map relating types and proofs of a syntactic source calculus to their counterparts in a calculus for meaning assembly. The Lambek-Van Benthem calculus, i.e. Lambek Calculus with Permutation, in retrospect MILL, has been an obvious choice for this semantic target calculus.

In natural languages, one easily finds cases of ostensible mismatches between the composition of form and meaning. I compare the strengths and weaknesses of two extensions of Lambek Calculus that purport to address these mismatches.

The first extension adds unary type-forming operations $\diamondsuit, \Box$ ('modalities') to the binary operations $/, \otimes, \backslash$ of Lambek Calculus. Residuation principles hold together both the 1-ary and 2-ary families. A powerful feature of the unary operations is their ability to replace *global* structural rules of Associativity/Commutativity by *controlled* versions, thus avoiding overgeneration of the syntactic source calculus. A more problematic aspect, especially of multimodal generalisations $\diamondsuit_i, \Box_i$, is the fact that they allow for the formulation of *construction-specific* structural rules. In Lambek's original setup, a grammar is the combination of a universal type logic and a language specific lexicon. With construction-specific rules, this attractive property is lost.

In the second part of the talk I report on joint work with Bernardi, Kurtonina, Moot, Bastenhof, Greco and others on Grishin's generalisation of Lambek calculus which introduces *dually* residuated operations $\oplus, \oslash, \obslash$ (coproduct, right and left difference) in addition to $\otimes, \backslash, /$ (product, left and right implication), and possibly interaction principles relating these two families (linear distributivities). The syntactic front end now takes the form of a focused display sequent calculus where formula polarities restrict the applicability of inference rules. The target calculus is $\text{MILL}_{\otimes,\cdot^\perp}$ with multiplicative conjunction and restricted linear implication $\cdot \multimap \perp$ w.r.t. a response type $\perp$. The Lambek-Grishin approach avoids complications of the syntactic source calculus that are motivated solely

by semantic considerations. Different scope construals of generalized quantifier expressions ('everyone', 'nobody', 'some politician', etc) are a case in point. On the downside, the generality of the (dual) residuation principles makes it difficult to limit the combinatorial possibilities of scope construal to what is actually observed in natural language semantics.

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Kleene Algebras and Morita Equivalence

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30 June 2025

Keywords:
Kleene module, Kleene algebra, Morita equivalence.

Kleene algebras were introduced by Kleene [4] as an axiomatization of regular events, with the intended application domain initially being biological systems. Conway, Parikh, Red'ko, Salomaa, and others continued this investigation (see for instance [1, 7, 8, 9]), and Kozen [5] provided a first-order axiomatization. The standard interpretation of Kleene algebras in computer science regards elements as programs and algebraic operations as control structure, but numerous other interpretations have proved fruitful.

Because Kleene algebras are analogous to rings, it is natural to consider Kleene modules—more particularly left-, right-, and bimodules—and indeed this has been done [2], though as with Kleene algebras the axiomatization is not fully agreed-upon. We make some observations about Kleene modules which we expect will be useful in further investigations. In ring theory the Morita category is of great interest, because Morita equivalence (defined as isomorphism in the Morita category) provides a precise sense in which two rings can have "the same" representation theory (note that representations of a ring are, in particular, modules). Here the Morita category is the category of rings with bimodules as morphisms and tensor product of bimodules as composition. This is an enrichment of the usual category of rings, because each homomorphism between rings induces a unique bimodule. Morita equivalence for semirings has been previously considered [3], but we are not aware of applications to Kleene algebras. This is where we make our contribution. The category of Kleene bimodules has tensor products and hence there is a Morita category of Kleene algebras. Moreover, the basic properties of the classical Morita category continue to hold for the Kleene-Morita category, and in particular if Kleene algebras K and S are Morita-equivalent then one is a matrix algebra over the other of a particularly simple form, identical to the situation for rings. This takes

on additional significance in the context of Kleene algebras and automata, for automata are matrices over Kleene algebras, and we see that Morita-equivalent Kleene algebras have equivalent categories of automata.

Turning to more specific (potential) applications, propositional dynamic logic and many of its variants induce Kleene modules over a Boolean algebra via the Lindenbaum-Tarski algebra of the logic, and Morita-equivalent Kleene algebras have equivalent categories of modules and hence yield equivalent algebraic semantics for propositional dynamic logic over a fixed Boolea algebra.

It is a general fact that, provided the scalars are the same on both sides, a bimodule over a ring can be extended to a ring via the tensor algebra construction. We observe that this carries over to Kleene algebras, with the asterate being added by taking the *-completion of the tensor algebra, and call the result a tensor Kleene algebra. Hence given a Kleene bimodule which is a Boolean algebra, one can canonically construct a Kleene algebra with tests—as defined by Kozen [6]—by extending this bimodule to its tensor Kleene algebra. Unfortunately this does not seem to be generally applicable to Lindenbaum-Tarski algebras because they are not naturally Kleene bimodules unless the underlying Kleene algebra is commutative. Nevertheless, the tensor Kleene algebra construction shows great flexibility with respect to the underlying logic, and so the challenge of devising appropriate right actions on Lindenbaum-Tarski algebras for dynamic logics becomes more urgent. To illustrate this flexibility, note that if we start with a Kleene bimodule which is a Heyting algebra, then the tensor Kleene algebra construction yields an analogue of a Kleene algebra with tests where the tests take values in a Heyting algebra. Since Heyting algebras provide algebraic semantics for constructive logic this could be very useful for modelling programs which may not terminate. Moreover there is a great variety of extra structure with which Kleene bimodules can be endowed, for instance MV algebras to reason about multiple truth values in database systems or for fuzzy reasoning about continuous systems, or orthomodular lattices to reason about quantum algorithms. The possibilities are endless and we are excited to continue exploring them.

Acknowledgment

The research for this project was completed under the advisorship of Professor Dexter Kozen as a project for his course at Cornell University on the subject of Kleene algebra taught in the Spring semester of 2024.

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Algebraic semantics for interpretability logics

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Keywords:
Modal logic, interpretability logic, algebraic semantics, completeness.

Interpretability logic **IL** extends provability logic **GL** with a new binary operator $\triangleright$. One of the most notable semantics for **IL** is Veltman semantics (see [3]), which extends Kripke frames with a family of relations $\{S_w : w \in W\}$ satisfying some properties.

It is known that the logic **IL** is complete with respect to Veltman semantics, but same cannot be said about its extensions, some of which have been proven to be incomplete (see [2]).

In this talk, we will define a new class of boolean algebras, which may be used to model interpretability logics. These algebras extend modal algebras (see e.g. [1]) in a natural way and may be considered to be generalizations of Veltman frames. Additionally, any consistent extension of **IL** is sound and complete with respect to this semantics.

Acknowledgments

This work is supported by Croatian Science Foundation under the project HRZZ-IP-2024-05-3882.

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Goedel's ontological proof

Zvonimir Šikić

Kurt Goedel worked for years on his ontological proof. He showed it for the first time to Dana Scott in early 1970. Fearing that his end was near, he wanted to make sure that what he proved will not disappear with him. In August 1970, when he felt significantly better, he said to Oscar Morgenstern that he hesitated to publish the proof, although he was satisfied with it, for fear that people would think he believed in God - because he was only logically investigating whether such a proof is possible with appropriate axiomatization. We present Goedel's axiomatization and the proof itself, and look critically at his axioms. We also present two axiomatizations that bring gods closer to the gods worshiped by existing religions.

Concurrent Rules Machines: a Model of Open Cyberphysical Systems

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Cyberphysical systems interact with their environment in complex ways. In addition to exchanging information with other systems, they affect and are affected by their physical / natural environment. Their components run concurrently and may be physically distributed, participating in both synchronous and asynchronous interactions.

Modeling interactions of a system with its environment presents a special challenge in open cyberphysical systems where the environment is nature. Existing approaches typically model the environment as yet another actor or component in the system. However, in cases of physical interactions, the unpredictability of this environment and the complexity of the physics involved means that "environment as a component" may not be a suitable approach.

Concurrent Rules Machines (CRMs) is a model for formal specification and analysis of open, distributed cyberphysical systems [1]. The CRM model makes interaction with the environment explicit and offers an algebra of composition and decomposition for construction and analysis of systems through their constituent components. Systematic and automatic verification of properties of systems modeled as CRMs poses a significant challenge. The mathematical presentation of the CRM semantics suggests a natural symbolic representation as a basis to model the inherently continuous properties of physics, the discrete nature of control actions, and representing environment effects.

In this talk we review the structure and operational semantics of the Concurrent Rules Machine (CRM) model and its algebra. We also describe symbolic execution as a means of reasoning about behaviors of CRM models. A symbolic execution represents possibly infinitely many executions and can focus on environments meeting requirements such as physically reasonable actions. Symbolic execution is sound, and is complete for a natural class of CRMs. We illustrate CRM concepts with a collection of simple cyber-physical system examples.

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Natural Deduction Systems for Conjunctive Multiple-Conclusion Logics

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Keywords:
Consequence relation, multiple-conclusions, 3-valued logic, intuitionistic logic, natural deduction, completeness.

On the disjunctive reading of multiple-conclusion consequence relations, a set of conclusions Y is said to follow from a set X of premises if at least one formula in Y is true whenever all formulae in X are true. Many properties of disjunctive multiple-conclusion logics are known. (Smiley [4] is the classic monograph on the subject. See also [5] for an explicit characterization theorem).

On the conjunctive reading of multiple-conclusion consequence relations, a set of formulas Y is said to follow from a set X of premises if all formulae in Y are true whenever all formulae in X are true. We consider classical and various non-classical (mainly three-valued and intuitionistic logics) conjunctive multiple-conclusion logics. After a brief discussion of the semantic characterization of conjunctive multiple-conclusion logics, we present sequent-style natural deduction systems for them. Finally, we deal with the problem of completeness for the systems considered.

The natural deduction system for classical conjunctive multiple-conclusion logic is based on a generalization of the sequent calculus in [2]. For the natural deduction system for conjunctive 3-valued multiple-conclusion logic we generalize the natural deduction system presented in [1]. To develop our system of natural deduction for conjunctive multiple-conclusion intuitionistic logic we use the idea presented by De Paiva and Pereira who [3] use the idea of indexing formulas to keep track of dependency of conclusions on premises.

Acknowledgment

This work is supported by Anadolu University under the project SBA-2024-2676.

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Properties of categorial grammars with k-type assignment

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Keywords:
Categorial grammars, k-valued grammars.

The notion of categorial grammars is a logical formalism for the representation of natural and synthetic languages. *AB-grammars* (named after K. Ajdukiewicz and Y. Bar-Hillel) were the first variant of categorial grammars, dating back to Ajdukiewicz's work [1] from 1935. In its modern form, this concept appears in Bar-Hillel's work [2]. Later, in the work [3], the fundamental properties of the formalism were established, one of which being the equality between the class of generated languages and the class of context-free languages without the empty word. This formalism plays a key role, since most other categorial grammars are its extensions through the addition of new operations and rules.

In categorial grammars, each symbol of the alphabet Σ is assigned an arbitrary number of categories, constructed from a certain set of "primitive" ones using two operations — right and left division. A word belongs to the generated language if there exists a choice of categories such that the corresponding string reduces to some chosen resulting category. Consider the phrase: "Ivan meets Maria". The syntactic units "Ivan" and "Maria" are assigned the type of noun phrase np. The verb "meets" is assigned the type $(np\backslash(S/np))$, where S is the type of a well-formed sentence. Thus, for the words in this phrase, there exists a type assignment $np; (np\backslash(S/np)); np$ such that the corresponding implication is derived: $np; (np\backslash(S/np)); np \Rightarrow S$.

In most variants of categorial grammars, a single element of the alphabet Σ may be assigned multiple distinct types. From a linguistic perspective, this property corresponds to the phenomenon of homonymy. Consider the sentence "A bear can bear a bear". In this sentence, the word "bear" must be assigned several different categories.

AB-grammars with at most k categories assigned to each symbol were introduced earlier in works by Kanazawa [4], Buszkowski [5], etc. For any fixed k, Kanazawa demonstrated that the class of languages generated by such grammars is learnable according to Gold's theory [6]. Specifically, there exists a

learning algorithm that converges to the correct grammar for any infinite input sequence drawn from the language. In opposite the class of context-free languages are not learnable which was proved by Gold [6]. Detailed information on these grammars can be found in the survey [7]. For these grammars we consider the case where only one category is assigned to each symbol.

We introduce a subclass of AB-grammars where where non-primitive categories are not allowed under divisions. Hence, categories like $p/(p/p)$ are not allowed in assignments. We call such grammars *basic categorial grammars* (BCG). For BCG we consider the case of assigning k categories to each symbol and denote the class of such grammars by $\mathcal{G}^k$. Classes of languages defined by such grammars form a hierarchy based on the number k. We prove the following algorithmic properties for class $\mathcal{G}^1$.

Theorem 1 *The problem of determining for arbitrary grammars G_1 and G_2 from the class $\mathcal{G}^1$ whether the language $L(G_1) \cap L(G_2)$ is infinite is algorithmically undecidable.*

Theorem 2 *The problem of determining for arbitrary grammars G_1 and G_2 from the class $\mathcal{G}^1$, where the number of symbols in the alphabets of both grammars is 11, whether the language intersection $L(G_1) \cap L(G_2)$ is empty, is algorithmically undecidable.*

The proof of undecidability is based on a reduction from a known undecidable problem. In this case, the problem used is checking the existence of a solution for an arbitrary Post correspondence system. This theorem strengthens a previously known result from Foret's work [8], which proved a similar theorem for AB-grammars, but required an alphabet with a significantly larger number of symbols.

The next part of the talk is devoted to encodings of any context-free language using AB-grammars with unique category assignment and grammars from the class $\mathcal{G}^2$. This is done in the sense of Greibach's hardest language theorem [9]. We use the following notation.

Definition 1 *The language $L_1 \subseteq \Sigma^*$* reduces by homomorphism *to the language $L_2 \subseteq \Omega^*$ ($L_1 \leq L_2$), if there exists a homomorphism $h\colon \Sigma \to \Omega^*$ such that a word w belongs to L_1 if and only if $h(w)$ belongs to L_2.*

For the class $\mathcal{G}^2$ we prove the following theorem, which states that any context-free language without the empty word can be encoded by some grammar from the class $\mathcal{G}^2$. Moreover, this encoding holds in both directions.

Theorem 3 *For an arbitrary context-free language $L \subseteq \Sigma^+$, there exists G — a grammar from the class $\mathcal{G}^2$ such that $L \leq L(G)$ and $L(G) \leq L$.*

For class of AB-grammars it is possible to show that such grammars can define a homomorphic encoding of any context-free language.

Theorem 4 *For an arbitrary context-free language $L \subseteq \Sigma^+$, there exists a categorial grammar with unique type assignment G such that $L \leq L(G)$.*

In particular, one of the consequences of this theorem is the solution to the problem posed in Foret's paper [8].

Theorem 5 *The problem of determining, for AB-grammars with unique category assignment G_1 and G_2, whether $L(G_1) \subseteq L(G_2)$, is algorithmically undecidable.*

Acknowledgment

The research reported in the talk is based on the PhD thesis of the author which is planned to be defended at the Steklov Mathematical Institute, under the supervision of Dr. Stepan L. Kuznetsov.

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On approximating semicomputable continua

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By the definition, each computable set in a computable topological space is indeed semicomputable, but the converse does not hold; that is, there exist semicomputable sets that are not computable. Nevertheless, certain topological properties of a set S may ensure that the implication

$$S \text{ semicomputable } \Rightarrow S \text{ computable}$$

holds. Since this is still not true in general, a natural question arises:

Under what conditions can a semicomputable set S be approximated by a computable subset to any given precision?

It is known that any semicomputable continuum S in a computable topological space can be approximated, with arbitrary precision, by a computable subcontinuum, provided that S is both chainable and decomposable.

Here we show that the decomposability condition can be replaced by assumption that S is **chainable from a to b**, where a is a **computable point**.

We have proven the following:

Theorem 1. *Suppose $(X, \mathcal{T}, (I_i))$ is a computable topological space. Let S be a semicomputable set in this space such that S is a continuum chainable from a to b, where a is a computable point. Then, for each open cover $\mathcal{U}$ of $(X, \mathcal{T})$, there exist a computable point $\hat{b} \in S$ and a continuum $\hat{S}$, chainable from a to $\hat{b}$, such that*

$$\hat{S} \subseteq S, \quad \hat{S} \approx_{\mathcal{U}} S,$$

and $\hat{S}$ is a computable set in $(X, \mathcal{T}, (I_i))$.