Time-Bounded Resilience

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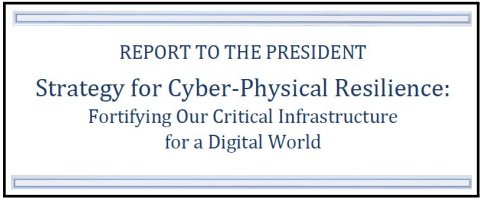
Resilience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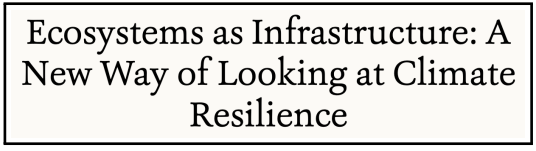
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Resilience

What is resilience?

"[Resilience emphasizes] the ability of a system to adapt and respond to change (both environmental and internal)." Bloomfield et. al., [2].

Why resilience?

"We must recognize the trade-off between efficiency and resilience. It is time to develop the discipline of resilient algorithms." Moshe Vardi, [1].

Overview

- Timed multiset rewriting (MSR) systems are an expressive formalism for modeling planning scenarios with discrete time.

- Expository example:
  1. **Example**: a researcher is planning travel to a conference.
  2. The researcher wants a **resilient** travel plan which achieves his goal despite issues such as flight delays.

- We will formalize resilience for planning scenarios based on timed MSR systems.

- At the end, we will discuss our Maude implementation of this example.

Resilience via Timed Multiset Rewriting Systems

- We want to model a planning scenario.

- High level idea:
  1. We represent states of the scenario via **configurations**.
  2. **Rewrite rules**, representing "actions" in the scenario, modify configurations.
     - **System rules** represent actions of our "protagonist."
     - **Update rules** can be seen as actions of an "adversary."
  3. **Planning** corresponds to finding **compliant** traces to a **goal** configuration.
  4. *n*-**Resilience** is a decision problem: can we find a compliant trace to a goal configuration which is resilient to *n* adversarial disruptions?

- There is an intuitive game-theoretic interpretation to this formalism: its complexity lands naturally within the polynomial hierarchy (PH).

First-order Formulas and Facts

- We fix a first-order alphabet Σ.

- **Atomic formulas** are of the form $R(t_1, \ldots, t_n)$, where
  1. R is an n-ary relation symbol in Σ, and
  2. the t_i are Σ-terms which may contain variables.

- **Facts** are atomic formulas without variables.

- **Timestamped atomic formulas** are of the form $F@(T + D)$, where F is an atomic formula, T is a **time variable**, and D is a natural number.

- **Timestamped facts** are of the form $F@t$, where F is a fact and t is a natural number.

Configurations

- **Configurations** are multisets of timestamped facts.

- The **global time** of a configuration is given by the timestamp of a (unique) timestamped fact of the form Time@t.

$$\{\text{Time@}(3d\ 14{:}42),\ \underline{\text{Attended}}(\text{main}, \text{no})@0,\ \text{At}(\text{FRA}, \text{airport})@(3d\ 14{:}05),\ \underline{\text{Event}}(\text{main})@(5d\ 12{:}00),\ \text{Flight}_2(\text{FRA}, \text{DBV})@(3d\ 15{:}25)\}$$

- Note – configurations contain only **ground terms** (i.e., no variables).

Rewrite Rules

- Configurations are modified by **rewrite rules**.
- There is a special rule Tick which increments the global time by one:

$$\mathsf{Time}@T \longrightarrow \mathsf{Time}@(T+1)$$

- All other rewrite rules are **instantaneous**, unable to modify the global time.

Instantaneous Rules

- **Instantaneous rules** have the form

$$\overbrace{\mathsf{Time}@T, \mathcal{W}, F_1@T_1, \ldots, F_n@T_n}^{\textbf{Precondition}} \mid \mathcal{C} \longrightarrow \underbrace{\mathsf{Time}@T, \mathcal{W}, Q_1@(T+D_1), \ldots, Q_m@(T+D_m)}_{\textbf{Postcondition}}$$

$\mathcal{W}$	—	multiset of timestamped atomic formulas (the **side condition**)
$F_i@T_i$ & $Q_j@T_j$	—	timestamped atomic formulas
$\mathcal{C}$	—	a set of **time constraints** of the form $T_1 > T_2 \pm N$ or $T_1 = T_2 \pm N$

Rule Application: Travel Example

Modeling “taking a (two-hour) flight” with an instantaneous rule:

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$$\mathsf{Time}@T, \mathsf{Flight}_2(x_1, x_2)@T_1, \mathsf{At}(x_1, \mathsf{airport})@T_2, \mid \{T = T_1, T_2 + 30 \leq T\} \longrightarrow \mathsf{Time}@T, \mathsf{Flight}_2(x_1, x_2)@T_1, \mathsf{At}(x_2, \mathsf{airport})@(T + 120),$$

Rule Application: Travel Example

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Not applicable! $T \neq T_1$.

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Not applicable! $T \neq T_1$.

After 43 applications of Tick:

$$\{\mathsf{Time}@(3d\ 13{:}25), \underline{\mathsf{Attended}}(\mathsf{main}, \mathsf{no})@0,\ \mathsf{At}(\mathsf{FRA}, \mathsf{airport})@(3d\ 14{:}05),\ \underline{\mathsf{Event}}(\mathsf{main})@(5d\ 12{:}00), \mathsf{Flight}_2(\mathsf{FRA}, \mathsf{DBV})@(3d\ 15{:}25)\}$$

Rule Application: Travel Example

$\{\mathsf{Time@}(3d\ 15{:}25), \underline{\mathsf{Attended}}(\mathsf{main}, \mathsf{no})@0,\ \mathsf{At}(\mathsf{FRA}, \mathsf{airport})@(3d\ 14{:}05),$
$\underline{\mathsf{Event}}(\mathsf{main})@(5d\ 12{:}00), \mathsf{Flight}_2(\mathsf{FRA}, \mathsf{DBV})@(3d\ 15{:}25)\}$

Rule Application: Travel Example

$\{\text{Time@}(3d\ 15{:}25), \underline{\text{Attended}}(\text{main}, \text{no})@0,\ \text{At}(\text{FRA}, \text{airport})@(3d\ 14{:}05),$
$\underline{\text{Event}}(\text{main})@(5d\ 12{:}00), \text{Flight}_2(\text{FRA}, \text{DBV})@(3d\ 15{:}25)\}$

$x_1 \mapsto \text{FRA}$
$x_2 \mapsto \text{DBV}$
$T \mapsto 3d\ 15:25$
$T_1 \mapsto 3d\ 15:25$
$T_2 \mapsto 3d\ 14:05$

Rule Application: Travel Example

$\{\text{Time@}(3d\ 15{:}25), \underline{\text{Attended}}(\text{main}, \text{no})@0,\ \text{At}(\text{FRA}, \text{airport})@(3d\ 14{:}05),$
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x_1	$\mapsto$ FRA
x_2	$\mapsto$ DBV
T	$\mapsto 3d\ 15:25$
T_1	$\mapsto 3d\ 15:25$
T_2	$\mapsto 3d\ 14:05$

Rule instance :

$$\begin{aligned}&\text{Time@}(3d\ 15:25), \text{Flight}_2(\text{FRA}, \text{DBV})@(3d\ 15:25),\\ &\quad \text{At}(\text{FRA}, \text{airport})@(3d\ 14:05),\\ &\quad \mid \{3d\ 15:25 = 3d\ 15:25, 3d\ 14:05 + 30 \leq 3d\ 15:25\}\\ &\longrightarrow \text{Time@}(3d\ 15:25), \text{Flight}_2(\text{FRA}, \text{DBV})@(3d\ 15:25),\\ &\quad \text{At}(\text{DBV}, \text{airport})@(3d\ 15:25 + 120)\end{aligned}$$

Rule Application: Travel Example

$\{\text{Time@}(3d\ 15{:}25), \underline{\text{Attended}}(\text{main}, \text{no})@0,\ \text{At}(\text{FRA}, \text{airport})@(3d\ 14{:}05),$
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Rule instance :

$$\begin{aligned}&\text{Time@}(3d\ 15:25), \text{Flight}_2(\text{FRA}, \text{DBV})@(3d\ 15:25),\\ &\quad \text{At}(\text{FRA}, \text{airport})@(3d\ 14:05),\\ &\quad \mid \{3d\ 15:25 = 3d\ 15:25, 3d\ 14:05 + 30 \leq 3d\ 15:25\}\\ &\longrightarrow \text{Time@}(3d\ 15:25), \text{Flight}_2(\text{FRA}, \text{DBV})@(3d\ 15:25),\\ &\quad \text{At}(\text{DBV}, \text{airport})@(3d\ 15:25+120)\end{aligned}$$

$\{\text{Time@}(3d\ 15{:}25), \underline{\text{Attended}}(\text{main}, \text{no})@0,\ \text{At}(\text{DBV}, \text{airport})@(3d\ 17{:}25),$
$\underline{\text{Event}}(\text{main})@(5d\ 12{:}00), \text{Flight}_2(\text{FRA}, \text{DBV})@(3d\ 15{:}25)\}$

Timed MSR Systems

• A **timed MSR system** is a set $\mathcal{R}$ containing the Tick rule and some finite number of instantaneous rules.

• A **trace** of $\mathcal{R}$ rules from an "initial" configuration $\mathcal{S}_0$ is a sequence $\mathcal{S}_0 \longrightarrow \cdots \longrightarrow \mathcal{S}_n$ of configurations, where some instance of a rule $r \in \mathcal{R}$ applied to $\mathcal{S}_i$ yields $\mathcal{S}_{i+1}$.

• A **goal configuration specification** designates conditions for a configuration to be a **goal configuration**. It contains pairs of the form $\langle \mathcal{S}, \mathcal{C} \rangle$, where $\mathcal{S}$ is a multiset of timestamped atomic formulas and $\mathcal{C}$ is a set of time contraints.

• For example:

$$\{\langle \{\underline{\text{Attended}}(\text{main}, \text{yes}) @ T_1\}, \emptyset \rangle\}$$

Critical Configurations and Compliance

- A **critical configuration specification** describes when a configuration is "critical."

$$\{\langle \text{Time@}T, \underline{\text{Attended}}(\text{main}, \text{no})@T_1, \underline{\text{Event}}(\text{main})@T_2\}, \{T > T_2\}\rangle\}$$

- A trace is **compliant** if it does not contain any critical configurations.

- Critical configurations can be thought of as **safety violations**, while compliant traces are analogous to **safe traces**.

Toward Resilience

- To model resilience, we need a notion of actions which are under the control of the system, and disruptions which are imposed on the system.

- We model the former via **system rules**, and the latter via **update rules**.

- **Example (update rule)** — A flight is delayed by 30 minutes:

$$\mathsf{Time}@T, \mathsf{Flight}_D(x_1, x_2)@T_1 \mid \{T = T_1\} \longrightarrow \mathsf{Time}@T, \mathsf{Flight}_D(x_1, x_2)@(T + 30).$$

Definition (Planning Scenario, [3])

If $\mathcal{R}$ and $\mathcal{E}$ are sets of system and update rules, $\mathcal{GS}$ and $\mathcal{CS}$ are a goal and critical configuration specifications, and $\mathcal{S}_0$ is an initial configuration, then the tuple $(\mathcal{R}, \mathcal{GS}, \mathcal{CS}, \mathcal{E}, \mathcal{S}_0)$ is a *planning scenario*.

Simplifying Assumptions

For our complexity results, we assume

1. Bounded depth of function applications in terms in facts occurring in traces;
2. η-**simplicity**: there is a fixed bound η on the number of (first-order and time) variables allowed to occur in a pair $\langle \mathcal{S}_i, \mathcal{C}_i \rangle$ in $\mathcal{CS}$; and
3. All planning scenarios are progressing.

Definition (Progressing Planning Scenarios (PPSs))

A planning scenario is **progressing** if, for each rule $r \in \mathcal{R} \cup \mathcal{E}$,

1. r is balanced (i.e., the precondition and postcondition have equal cardinality),
2. r consumes only facts with timestamps in the past or at the current time, and
3. r creates *at least one* fact with timestamp greater than the global time.

Simplifying Assumptions

Proposition

For η-simple planning scenarios, the trace compliance problem is in P.

Given an appropriate ground substitution, we can verify in polynomial time in the size of an η-simple PPS $A = (\mathcal{R}, \mathcal{GS}, \mathcal{CS}, \mathcal{E}, \mathcal{S}_0)$ that:

- a configuration $\mathcal{S}$ (with the same number of facts as $\mathcal{S}_0$) is a **goal configuration** w.r.t. $\mathcal{GS}$.
- a configuration $\mathcal{S}$ (with the same number of facts as $\mathcal{S}_0$) is a **critical configuration** w.r.t. $\mathcal{CS}$.
- a **rule** r **is applicable** to $\mathcal{S}$, and whether or not $\mathcal{S}'$ is the result of this application.

Formalizing Resilience

Intuitive idea: The system will have a + b time units to achieve its goal, and update rules can only be applied in the first a time steps; the last b time steps are the recovery time afforded to the system.

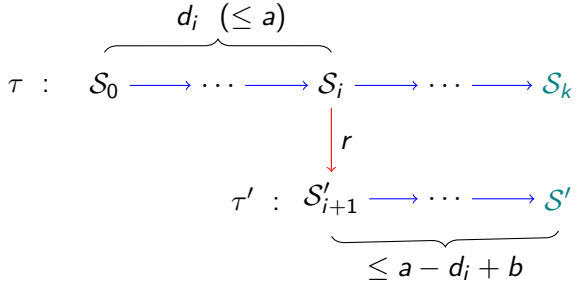


Figure: An (n, a, b)-resilient trace τ and an $(n-1, a-d_i, b)$-resilient reaction trace τ'. The horizontal arrows correspond to system rule applications, while the downward-facing arrow represents an update rule application. The configurations $\mathcal{S}_k$ and $\mathcal{S}'$ on the far right are goal configurations.

Formalizing Resilience

Definition (The (n, a, b)-resilience problem)

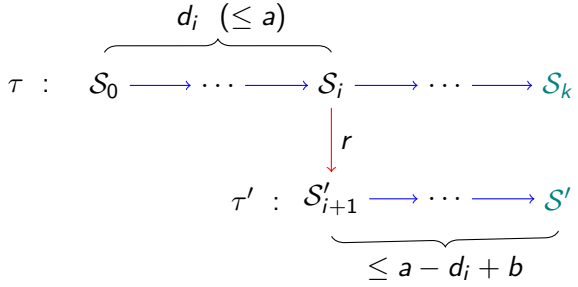
Let $a \in \mathbb{Z}^+$ and $b \in \mathbb{N}$. We define (n, a, b)-resilience by recursion on n. Inputs to the problem are planning scenarios $A = (\mathcal{R}, \mathcal{GS}, \mathcal{CS}, \mathcal{E}, \mathcal{S}_0)$. A trace is $(0, a, b)$-*resilient with respect to* A if it is a compliant trace of $\mathcal{R}$ rules from $\mathcal{S}_0$ to a goal configuration and contains at most $a + b$ applications of the Tick rule. For $n > 0$, a trace τ is (n, a, b)-*resilient with respect to* A if

1. τ is $(0, a, b)$-resilient with respect to A, and
2. for any system or goal update rule $r \in \mathcal{E}$ applied to a configuration $\mathcal{S}_i$ in τ, with $\mathcal{S}_i \longrightarrow_r \mathcal{S}'_{i+1}$, where global time t_i in $\mathcal{S}_i$ satisfies $d_i = t_i - t_0 \leq a$, there exists a *reaction trace* τ' of $\mathcal{R}$ rules from $\mathcal{S}'_{i+1}$ to a goal configuration $\mathcal{S}'$ such that τ' is $(n - 1, a - d_i, b)$-resilient with respect to $A' = (\mathcal{R}, \mathcal{GS}, \mathcal{CS}, \mathcal{E}, \mathcal{S}'_{i+1})$.

A planning scenario $A = (\mathcal{R}, \mathcal{GS}, \mathcal{CS}, \mathcal{E}, \mathcal{S}_0)$ is (n, a, b)-*resilient* if an (n, a, b)-resilient trace with respect to A exists. The (n, a, b)-resilience problem is to determine if a given planning scenario A is (n, a, b)-resilient.

Formalizing Resilience

Intuitive idea: The system will have a + b time units to achieve its goal, and update rules can only be applied in the first a time steps; the last b time steps are the recovery time afforded to the system.



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Formalizing Resilience

The reaction trace τ' can be interpreted as a change in the plan τ, made in response to an external disruption (*i.e.*, the system/goal update rule r) imposed on the system. Note that it is this "replanning" aspect of our definition that intuitively distinguishes it from the related notion of robustness.

In the Definition, the global time t' in $\mathcal{S}'$ satisfies $t' - t_0 \leq a + b$; *i.e.*, despite the application of n instances of update rules, an (n, a, b)-resilient trace reaches a goal within $a + b$ time units. Furthermore, observe that a trace is (n, a, b)-resilient with respect to a planning scenario A if and only if it is (n, a, b')-resilient with respect to A for all $b' \geq b$. Similarly, all (n, a, b)-resilient traces with respect to A are (n', a, b)-resilient with respect to A for all $n' \leq n$.

Complexity Results

Definition

A decision problem is in Σ_n^{P} (for n odd) if and only if there exists a polynomial-time algorithm M such that an input x is a *yes* instance of the problem if and only if

$$\exists u_1 \forall u_2 \exists u_3 \ldots \forall u_{n-1} \exists u_n \; M(x, u_1, \ldots, u_n) \text{ accepts,}$$

where the u_i are polynomially-bounded in the size of x.

In our case, the existentially-quantified variables represent compliant goal traces, while the universally-quantified variables represent update rule applications.

Complexity Results

Theorem

The (n, a, b)-resilience problem for η-simple PPSs with traces containing only facts of bounded size is $\Sigma^{\mathsf{P}}_{2n+1}$-complete.

Upper bound — by the quantifier-alternation characterization of $\Sigma^{\mathsf{P}}_{2n+1}$.

Lower bound — by a reduction from $\Sigma^{\mathsf{P}}_{2n+1}$-SAT.

Comments on Upper Bound

Even without assuming η-simplicity, the (n, a, b)-resilience problem for PPSs with traces with bounded nesting of function symbols is in $\Sigma^{\mathrm{P}}_{2n+3}$.

Given any such PPS, we allow each universal quantifier to range over an additional ground substitution, which is used in the verification algorithm to check that an arbitrary configuration in the preceding witness trace is noncritical.
This check can be done in polynomial time.
If this check succeeds for *all* configurations and *all* appropriate ground substitutions, then every witness trace is compliant.

Comments on Lower Bound

Even for 1-simple PPSs, the (n, a, b)-resilience problem is $\Sigma^{\mathrm{P}}_{2n+1}$-hard.

We show this by a reduction from Σ_{2n+1}-SAT, the language of true quantified Boolean formulas (QBF) with $2n+1$ quantifier alternations, where the first quantifier is existential and the underlying propositional formula is in 3-CNF form.

$$\psi := \exists x_1 \forall x_2 \exists x_3 \ldots \forall x_{2n} \exists x_{2n+1}\, \varphi(x_1, x_2, x_3, \ldots, x_{2n+1})$$

The truth of a QBF formula can be analyzed by considering the *QBF evaluation game* for the formula. Duplicator chooses assignments for existentially-quantified variables with the goal of satisfying the underlying Boolean formula, while Spoiler chooses assignments for universally-quantified variables with the goal of falsifying it.
A QBF ψ is true if and only if Duplicator has a winning strategy in this game.

Comments on Lower Bound

We compute a 1-simple PPS which is (n, a, b)-resilient if and only if ψ is true.
The computed 1-simple PPS has traces containing only facts of bounded size.
In our reduction, we encode the positions of this QBF evaluation game into configurations, where a position of the QBF evaluation game for a formula

$$\psi := \exists \overline{v}_1 \forall \overline{v}_2 \exists \overline{v}_3 \ldots \forall \overline{v}_{2n} \exists \overline{v}_{2n+1} \varphi(\overline{v}_1, \overline{v}_2, \overline{v}_3, \ldots, \overline{v}_{2n+1})$$

is a sequence $\mathcal{P} = V_1, \ldots, V_j$ of assignments to the variables in $\overline{v}_1, \ldots, \overline{v}_j$, $j \leq 2n+1$.
If j is even, the position $\mathcal{P}$ *belongs to Duplicator*; otherwise, it *belongs to Spoiler*.
The player who owns a given position makes the next move, choosing an assignment for the variables in the tuple $\overline{v}_{j+1}$. We use system rules to model assignments made by Duplicator, while update rules are used to model assignments made by Spoiler.
Intuitively, the goal configurations are those positions of the game which encode assignments satisfying the underlying formula φ.

Comments on Lower Bound

This encoding does not depend on the parameters a and b of the resilience problem:

the computed scenario admits an $(n, 1, 0)$-resilient trace
if and only if
it admits an (n, a, b)-resilient trace for all $a \in \mathbb{Z}^+$ and $b \in \mathbb{N}$.

It follows easily from the simulation of the QBF evaluation game that:

for all $a \in \mathbb{Z}^+$ and $b \in \mathbb{N}$, the computed scenario is (n, a, b)-resilient
if and only if
Duplicator has a winning strategy for the QBF evaluation game for the formula ψ.

Travel Planning in Maude

- The goal is to attend a set of events in different places, with some required and some optional.
- There is a knowledge base of flights to choose from.
- Updates include
  1. flight delay, cancellation, or diversion; and
  2. change of event start or duration.
- A critical configuration is one where the current time is later than the start time of a required event and the event has not been attended.

RWL vs MSR

- MSR and RWL both model systems using a notion of state and rewrite rules.
- In both cases the semantics is given by rewrite traces.
- MSR is better suited for analysis of general problems (classes of models) – decidability, complexity ...
- RWL is better suited for prototyping and automated verification of specific models.

How do RWL models differ from MSR models?

- RWL system state is represented by data structures rather than facts.
- RWL provides a mechanism for specifying these data structures and operations on them (Equation theories)
- In Real Time RWL timers, delays, durations control the passing of time rather than uniform unit ticks, an optimization avoiding unnecessary tick rule applications.
- In our case study, the passage of time is controlled using event/action duration:
  1. taking a flight takes time, and
  2. searching for a flight is instantaneous.

Some details — state representation

`tc(dateTime,city,location,events)` — planning
`tc(dateTime,city,location,events, event, flightLists)` —traveling

- Maude Example

```
dateTime = dt(yd(23, 247), hm(12, 42)),
city = FRA, location = airport
event = ev("id215", DBV, center, yd(23,249), hm(14,0), hm(120,0),
false) attendance optional
flightList(s) = fi(fl(FRA,DBV,"id14",hm(15,25),hm(2,0))) –abstract flight
dt(yd(23,247),hm(15,25)), — departure date time
dt(yd(23,247),hm(17,25)) — arrival date time
```

- MSR example

$$\{\text{Time@}(3d\ 14{:}42), \underline{\text{Attended}}(\text{main}, \text{no})@0,\ \text{At}(\text{FRA}, \text{airport})@(3d\ 14{:}05),\ \underline{\text{Event}}(\text{main}, \text{id}_{215})@(5d\ 12{:}00), \text{Flight}_2(\text{id}_{14}, \text{FRA}, \text{DBV})@(3d\ 15{:}25)\}$$

Some details — rules

- `plan` – find flight lists to next event location
- `flt` – take flight, duration = arrival - current time
- `event` – attend event, duration = event dur + time to airport
- `fltDigress` — apply a flight update
- `replan` – when current time is too late for event or flight

Checking n,a,b-resilience — input

initial state — a planning state with the full set of events to attend
tc(dateTime,city,location,events)

critical state – tcCrit(....)

goal state – tc(dateTime,city,location,mtE) – a terminal state

Checking (n, a, b)-resilience — search algorithm

- Search algorithm
  1. Use Maude search to find a compliant trace. If n $= 0$ return true.
  2. Step through this trace. At each point make a branch for each enabled update, decrement n and go to 1.

- How to do (2):
  1. Convert the trace found by Maude search into a sequence of rule instances.
  2. For each prefix, a maude strategy, and each update, append the update rule and use srewrite to find all updates for this point in the trace.

Experimental results

N:	1		2		3	
2ev	**R?**	**time**	**R?**	**time**	**R?**	**time**
247	N	86ms	-	-	-	-
246	Y	81ms	Y	147ms	N	7476ms
3ev	**R?**	**time**	**R?**	**time**	**R?**	**time**
247	N	1400ms	-	-	-	-
246	Y	325ms	Y	685ms	NF	-

(a) flight/system update rules

N:	1		2		3	
2ev	**R?**	**time**	**R?**	**time**	**R?**	**time**
247	Y	78ms	N	77ms	-	-
246	Y	98ms	N	34800ms	-	-
3ev	**R?**	**time**	**R?**	**time**	**R?**	**time**
247	Y	143ms	N	2627ms	-	-
246	Y	220ms	Y	633ms	Y	2634ms

(b) event/goal update rules

Figure: Summary of (n, a, b)-resilience experiments

Summary

We have:

1. Described timed MSR systems for modeling planning scenarios.
2. Given a formal definition of resilience and analyzed its complexity.
3. Implemented this formalism in Maude and run experiments on resilience of our travel example.

Questions?

References I


M. Vardi, "Efficiency vs. resilience: What covid-19 teaches computing," *Communications of the ACM*, vol. 63, no. 5, pp. 9–9, 2020.


R. Bloomfield, G. Fletcher, H. Khlaaf, P. Ryan, S. Kinoshita, Y. Kinoshita, M. Takeyama, Y. Matsubara, P. Popov, K. Imai, *et al.*, "Towards identifying and closing gaps in assurance of autonomous road vehicles–a collection of technical notes part 1," *arXiv preprint arXiv:2003.00789*, 2020.


M. A. Alturki, T. Ban Kirigin, M. Kanovich, V. Nigam, A. Scedrov, and C. Talcott, "On the formalization and computational complexity of resilience problems for cyber-physical systems," in *Theoretical Aspects of Computing–ICTAC 2022: 19th International Colloquium, Tbilisi, Georgia, September 27–29, 2022, Proceedings*, pp. 96–113, Springer, 2022.

References II

T. Ban Kirigin, J. Comer, M. I. Kanovich, A. Scedrov, and C. L. Talcott, "Time-bounded resilience," in *Rewriting Logic and Its Applications - 15th International Workshop, WRLA 2024, Luxembourg City, Luxembourg, April 6-7, 2024, Revised Selected Papers* (K. Ogata and N. Martí-Oliet, eds.), vol. 14953 of *Lecture Notes in Computer Science*, pp. 22–44, Springer, 2024.

T. Ban Kirigin, J. Comer, M. Kanovich, A. Scedrov, and C. Talcott, "Technical report: Time-bounded resilience," *arXiv preprint arXiv:2401.05585*, 2024.