Concurrent Rules Machines:

A Model of Open Cyberphysical Systems

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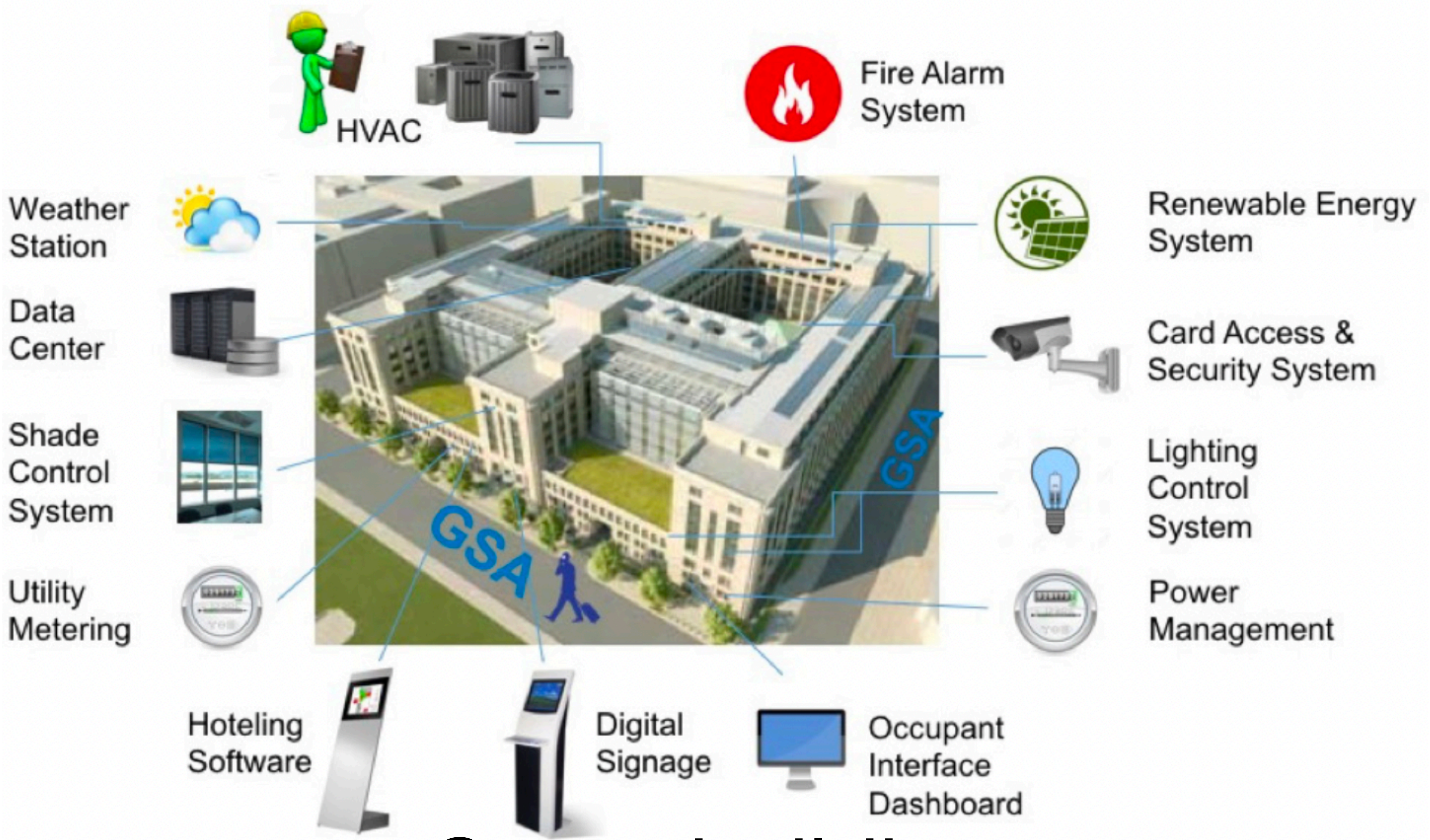
The problem

Cyberphysical systems (CPS) are increasingly present, playing critical roles.

Precision agriculture

Waymo

Smart city



Smart building

CPSs operate in unknown, unpredictable environments: effecting and being effected.

Requirements

It is important to model and reason about interaction with the environment at design time.

A foundational formal framework for developing CPS models should support representation and reasoning about:

- interaction with unknown/unpredictable environments
- discrete and continuous change (cyber and physical)
- concurrent and distributed execution
- diverse interactions among components including synchronous and asynchronous discrete interactions, as well as continuous interactions such as flow of a physical quantity among components (e.g., force, energy, liquid, etc.)
- composition operations that model these different interactions
- methods to support scaling in time and space such as symbolic reasoning and abstraction; and compositional design and reasoning

State of the Art (a sketch)

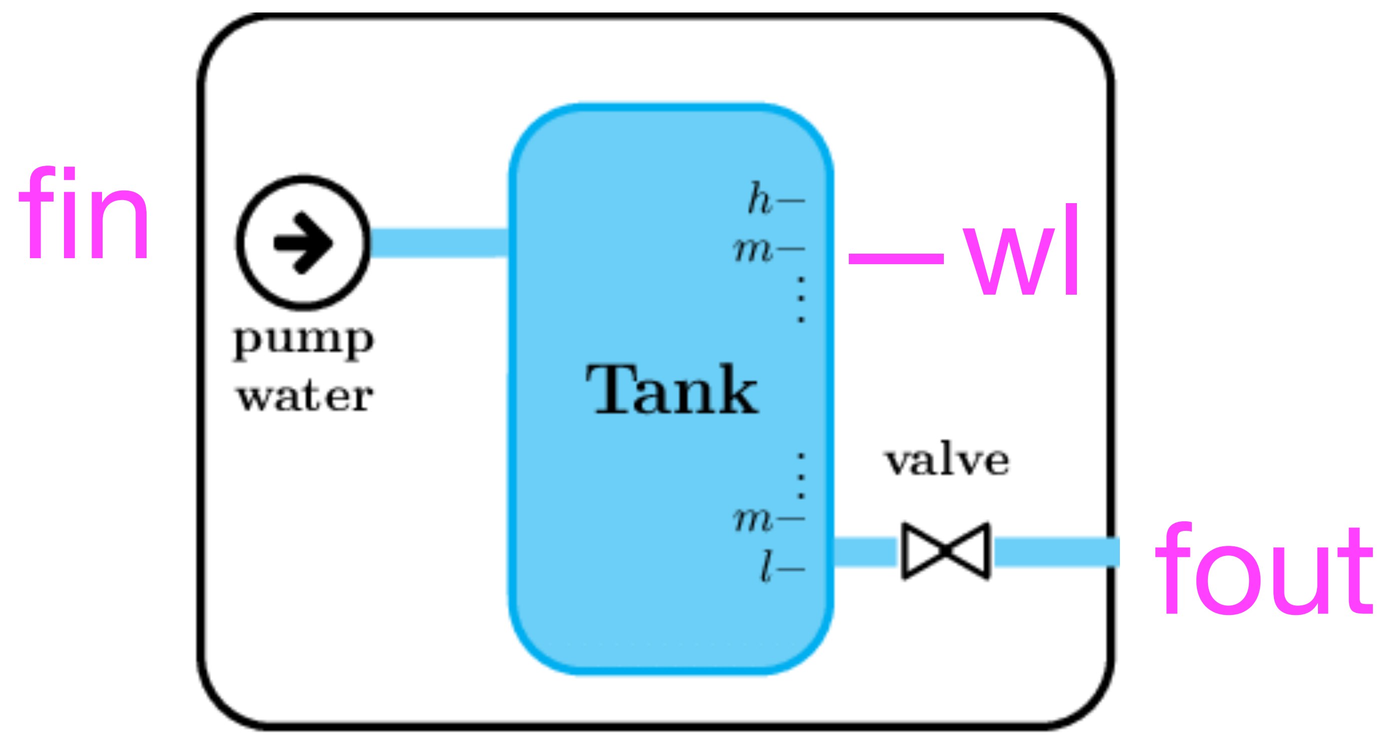
- Many kinds of automata (Team Automata, Constraint Automata, IO Automata, Interface Automata, ...) and process algebras: concurrent execution and composition with a mix of synchronous and asynchronous interaction with other components
- Rewriting logic -- a formalism for specifying concurrent and distributed systems. Rich algebra of modules for composition, diverse reasoning tools. Native execution model is closed system. Symbolic execution captures some enviornment effects
- Composable Semantics — behaviors as sets of Timed (I/O) Event Stream, composition parameterized by interaction constraints.

Contributions

- Definition of the Concurrent Rules Machine (CRM) model: structure and operational semantics.
  - Interaction with the environment is explicit, enabling representation and reasoning about open systems
  - Supports multiple models of time including implicit time (before/after relations)
- An algebra of CRM components including several composition operations, and a division relation for reasoning about decomposition.
- Symbolic execution — a step towards executability and automating reasoning

Running Example

- A Water tank with components:
  - WaterLevel (WL) -- water level sensor
  - Control (WCI/WCO) --- controls in and/or out flow
- Physical constraints:
  - $WLMin <= wl <= WLMax$
  - fin, fout in [0,1] — in/out flow rates
  - $wl(t) = wl(t0) + (fin - fout) * (t - t0)$
- Controller goal
  - $WLmin < WLMinS <= wl <= WLMaxS < WLMax$



Concurrent Rules Machines in a Nutshell

CRM Structure

Key concepts

- D -- data values
- Name -- variable names
- Valuation -- finite map, V, from Name to D
- Guard -- predicate on values of names, V |= G
- Action -- evaluates to a valuation, view as a valuation update
    - non-conflicting action sets
- Rule -- (guard, action set)

A CRM is given by a tuple (I,A_0,R,T) where

- I = (Imp, Exp, Sh) -- interface == disjoint sets of names
    - Env writes Imp -- imports
    - CRM writes Exp -- exports
    - both write/read Sh -- shared
- A_0 is a finite set of (non-conflicting) actions for initialization
- R is a finite set of Rules
- T is a finite set of (non-conflicting) actions for step termination

WL CRM

```
  WL = (WL.I, WL.A0, WL.R, WL.T)
where
  WL.I = ({'fin, 'fout, 'time}, {'wl},{})
  --- the interface with imports ('fin 'fout 'time), exports ('wl)
      and no shared variables ({}).
  WL.A0 = {(< >, \lambda (). 'wl := WCminSafe)}
     --- the initialization action
  WL.R = { (true, {a.wl})}
     --- the rule updating the water level
  WL.T = {(< 'time, 'fin, 'fout >,
          \lambda (t, fin, fout).
              {'t0 := t,'fin0 := fin, 'fout0 := fout} )}
         --- the termination action that caches the
             values of 'fin 'fout and 'time
 and
  a.wl = (< 'fin0,'fout0,'t0,'time , 'wl >,
           \lambda (fin0, fout0, t0, t, wl0).
               'wl := wl0 + (fin0 - fout0) * (t - t0) )
```

CRM Semantics

- CRM execution state is a valuation function: V : Names -f> D
- Execution alternates between action of Env and CRM rules
  - init: empty -env> V' -A_0-> V'' -T> V_0
  - step: V_j -env> V'_j -rs-> V''_j -T> V_{j+1}

where

  - V_j -env> V'_j -- Env updates some of Imp + Sh
  - V'_j -rs> V''_j -- action of rs, a set of rules st guards are satisfied by V'_j and combined actions are non-conflicting in V'_j
  - V''_j -T-> V_{j+1} -- application of actions in T
- Traces are sequences of steps

WL execution

```
time fin   fout  wl    t0  fin0  fout0
  0   1    .5                              Env initiates
                 2.0   0     1       .5    WL A0;T
  1   0    1                               Env writes
                 2.5   1     0      1      WL,R ; WL.T
  2   1    0                               Env writes
                 1.5   2     1      0      WL,R ; WL.T
  3   1    0                               Env writes
                 2.5   3      1     0      WL,R ; WL.T
  .....
```

CRM Algebra

- mtCRM = {mt,mt,mt,mt}
- C1 << C2 componentwise subset
- C1 U C2   componentwise union -- is CRM under some conditions
- C1 /\ C2 componentwise intersection -- CRM
- C1 x C2 product -- a restriction of U, guaranteed to be a CRM
- (Exists name)CRM -- hiding, makes name private
- C1 ; C2 -- a sequential composition, where at each step
  - the Env acts, then chosen rule set C1 is applied followed by T1, then chosen ruleset of C2 followed by T2
- C1 + C2 a sequential composition where at each step
  - the Env acts, then chosen rule set C1 is applied then the chosen ruleset of C2 followed by the combined actions of T1 and T2
- C1 divisibleBy C2 relation -- essentially C1 covers C2
- C1/C2 -- defined if C1 divisibleBy C2 -- essentially the set Q st C2 x Q ~ C1

CRM intersection, union, x, are associative, commutative, and idempotent.

WL division

WCO divides WCIO and WCI is in WCIO/WCO

```
      WCIO= (WCIO.I, WCIO.A0, WCIO.R, WCIO.T)
    where
      WCIO.I = ({'wl},{'fin, 'fout}, {}) --- imports, exports, shared
      WCIO.A0 = {(< >,\lambda (). 'fin :=  0; 'fout := .5)}
      WCIO.R =
       {  RI: {wc.r1, wc.r2}
          RO: { (true, {a.wo}) }
        }
      WCIO.T = {}

  WCO = (WCO.I, WCO.A0, WCO.R, WCO.T)
where
  WCO.I = ({'wl}, {'fout}, {}) -- imports,exports,shared
  WCO.A0 = {(< >,\lambda (). 'fout := .5)}
  WCO.R = { RO: (true,{a.wo}) }
  WCO.T = {}
```

What is missing?

- CRMs allow us to directly represent interaction of a system with its environment.
- One can choose different system boundaries and move components in and out using the CRM algebra.
- BUT
- How do we prove properties of specific CRMs?
- How can we automatically generate traces?
- How can we automate reasoning tasks when there is possibly unbounded non-determinism in the actions of the environment?

Idea

- Represent updates by the environment by fresh (typed) symbols (ala uninterpreted constants), possibly constrained to represent physically meaningful change or operational domain assumptions.
- CRM actions inherit the symbolic nature, with the results of actions represented by fresh symbols constrained to be equal to the action function applied to symbolic values.
- A constrained symbolic valuation represents the set of its ground instances.
- At each potential execution step we need to check that the accumulated constraints are satisfiable, i.e. the symbolic execution describes a non-empty set of actual executions.
- In this setting one can automate reachability analysis if a suitable constraint solver is available.

Some details I

- Execution state is a symbolic valuation: (*W,b*)
  - *W* maps names to symbols
  - *b* is a constraint (boolean expression) over these symbols
- The meaning of (W,b) is a set of concrete valuations is given by its instances.
  - Substitution σ is an instance of (*W,b*) if $b[\sigma]$ is true
  - V is an instance of (*W,b*) if $V = W[\sigma]$ $(= \sigma \circ W)$ for σ an instance of (W,b)

Some details II

- R is a candidate ruleset for (W,b) if
  - each rule guard is satisfied by every instance of (W,b)
  - the joint action sets of R is non-conflicting in every instance of (W,b)
- Effect of action set A on symbolic valuation
  - $A[[W]] = (Z^A, b^A)$
  - Z^A — maps write names of A to fresh symbols
  - b^A -- the conjunction of constraints for each action of A

Symbolic CRM step

$$
\begin{aligned}
(W, b) &\rightarrow_E (W^E, b \wedge b^E) \\
&\rightarrow_R (W^R, b \wedge b^E \wedge b^R) \\
&\rightarrow_T (W^T, b \wedge b^E \wedge b^R \wedge b^T)
\end{aligned}
$$

- E environment action
- R action by candidate rule set
- Termination action

Constraining the environment

- CRM steps are constrained to conform to the actions of cadidate rules and terminal actions
- The environment is constrained uniformly by a constraint specification *envB* = ($\underline{ns}$,*cs*) where *cs* is a set of assignments to and boolean expressions over names in $\underline{ns}$.
- The meaning of assignments in *envB* depend on the starting valuation and the updated valuation of a step:
    - $envB[[W^{E}(\underline{ns}),\mathrm{W}(\underline{ns})]]$
- W^{E} is *W* updated by fresh symbols for the names assigned by the environment.
- Note *envB* constrains not only the values assigned but also the actions of the environment

Example: Input controller

```
  WCI = (WCI.I, WCI.AO, WCI.R, WCI.T)

where

  WCI.I = ({'wl,'time}, {'fin}, { })  --- {Im,Ex,Sh}

  WCI.AO = {(< >, \lambda (). 'fin := finInit)}

  WCI.R = { (true, {ic.fin})}

  WCI.T = (< 'fin >, \lambda f. 'fin0 := f) --- caching 'fin

  ic.fin = (< 'fin0, 'wl >, \lambda (f0,  wl). finexp

  finexp = 'fin := (if wl > WL.smx then 0  else (if wl < WL.smn then 1 else 1/2 fi) fi) )

  envB = (<'wl 'fin 'fout 'time 't0 >,

          WL.min <= 'wl and 'wl <= WL.max and  'wl := 'wl + ('fin - 'fout) *  ('time - 't0) )

wlInit = 3   WL.max = 5  WL.min = 3  WL.smx = 9/2   WL.smn  = 7/2
```

Symbolic reasoning

- We consider two simple questions about WCI that can be answered using symbolic execution.
- Q1: Can the water level go above `WL.smx`?
- The answer is yes. (using symbolic search and SMT solver)

```
(v('fin,0) := 31/32) (v('fout,0) := 1/32) (v('wl,0) := 3)
(v('fin,1) := 1/2)   (v('fout,1) := 1/16) (v('wl,1) := 63/16)
(v('fin,2) := 1/2) (v('fout,2) := 1/8) (v('wl,2) := 35/8)
(v('fin,3) := 0)   (v('fout,3) := 1/2) (v('wl,3) := 19/4)
```

Symbolic reasoning II

- Q2: Can the water level reach `WL.mx`?
- The answer to the second question is, up to time 12, no.
- However if we change the time step to be increments of 2, then the answer is yes.

```
(v('fin,0) := 1)  (v('fout,0) := 0 (v('wl,0) := 3
(v('fin,1) := 0)  (v('fout,1) := 1/2) (v('wl,1) := 5
```

Soundness and Completeness Theorems

- **Theorem 1.** (*Soundness*) Each instance of a symbolic trace of a CRM is a (concrete) trace of that CRM

- **Theorem 2** (<u>*Restricted Completeness*</u>). If CRM, *C*, is symbolically complete, and

$$Tr = V_j \rightarrow_{F_j,R_j} V_{j+1} \mid -1 \leq j < m$$

is a concrete trace of *C* such that F j satisfies *envB* for $-1 \leq j < m$, then there is a symbolic trace, $Tr*$, of *C* with environment constraint *envB* such that *Tr* is an instance of $Tr*$.

Symbolic Completeness

- **Definition 3** (*Symbolically complete CRM*). A CRM is symbolically complete if given any (reachable) symbolic valuation (*W,b*), and substitution, σ, that satisfies *b*; if rule set *R* is a candidate for execution in the context of *W*[σ] then *R* is a candidate for execution in the context of *W*[σ′] for any σ′ satisfying *b.*

- **Lemma 1** (*One (sub)step completeness*). If CRM, *C*, is symbolically complete, (*W,b*) a reachable symbolic valuation, σ, an instance of (*W,b*), *V* = *W*[σ] the corresponding valuation instance, and *R* a candiate rule set wrt *V* then

$$V \rightarrow_R V' \;\; implies \; (W, b) \rightarrow_A (W', b')$$

where *V′* is an instance of (*W′,b′*)

Summary and Future Work

- CRM is a model of concurrent (and distributed) computation that supports
  - representing interaction with arbitrary environment
  - discrete and continuous actions
  - compositional specification via the algebra of compositions and decomposition
- Symbolic execution is step twoards automated verification
  - it is sound and complete for large class of CP CRMs
- Future work
  - Implementation in RWL
  - Constrained CRM
  - Composition via interaction constraints modeling (a)synchrony, flow, space
  - Compositional reasoning

That’s all for now !
Questions?

Some References

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