

Abstract Socratic dialogue and dialectical logic

Teo Šestak

University of Zagreb
teo.sestak@fsb.unizg.hr

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Socratic dialogue

- a form of argumentative dialogue,
- questions and clarifications,
- is often described as dialectical.

 Vlastos, G., *The Socratic Elenchus*, Oxford Studies in Ancient Philosophy 1: 27–58 (1983).

- Socratic dialogue is of the form

$$\{C_1, \dots, C_n, P\} \vdash \neg P,$$

- commitments C_i are added to the set of commitments, until they suffice to refute a proposition P .

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Instead of adding commitments to a set, we abstract away and consider abstract information states (argumentation states).

$$W = \{w, v, u, \dots\}.$$

Socratic step of questioning and clarification allows Socrates and his interlocutor to go from one state to another. We formalize this via accessibility relation

$$R \subseteq W \times W.$$

Example

Let

$$W = \mathcal{P}(\mathcal{L}),$$

where $\mathcal{L}$ is a language of some theory. Furthermore, let wRv if and only if $v = w \cup \{\varphi\}$, where $\varphi \notin w$.

This gives us a (trivial, in a sense) model of questioning and clarification. We may add a formula (any formula we are not already committed to) to the set of commitments.

But Socrates does not only question in order to add a commitment to a set of commitments (or, in our case, change the information state). He sometimes concludes things, often based on the last added assumption (which to us is the last transition made).

Hence, given the transition $w \xrightarrow{R} u$, Socrates may pick another information state v , which is reachable from u , if we arrive at it from w . In other words, there is a ternary relation S such that $S(w, u, v)$.

$$\begin{array}{ccc} u & \overset{S}{\dashrightarrow} & v \\ \uparrow R & & \\ w & & \end{array}$$

Example

Extending our previous example, in which

$$W = \mathcal{P}(\mathcal{L}),$$

and R represents adding a new formula, we define S by $S(w, u, v)$ if and only if $v = u \cup \{\varphi\}$ for some φ such that $u \vdash \varphi$, but $w \not\vdash \varphi$.

Now Socrates has a way to derive $\neg P$, where P is his interlocutor's assertion, from the current set of commitments.

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We study structures (W, R, S) , where W is a non-empty set, $R \subseteq W \times W$ and $S \subseteq W \times W \times W$.

These are similar to Veltman frames, which model interpretability logics, but without constraints on R and S .

We want to use these in order to talk about dialectical contradictions. To each $w \in W$, we adjoin a set of propositional variables which hold in w .

These are interpreted as commitments in each step of the dialectical process and are extended to the set of formulas of propositional logic. We want to say that a formula φ (dialectically) contradicts ψ if accepting φ would allow Socrates to conclude $\neg\psi$, i.e., at $w \in W$

$$\varphi \not\vdash \psi \text{ if and only if } \forall u (wRu \ \& \ u \Vdash \varphi \rightarrow \exists v (S(w, u, v) \ \& \ v \Vdash \neg\psi)).$$

This gives rise to a logical system **DiaL**, whose language is given by the following:

$$\varphi ::= p \mid \perp \mid \varphi \rightarrow \varphi \mid \varphi \not\vdash \varphi,$$

where $p \in \text{Prop}$.

We also note that we may define modal $\Box$ operator as $\Box\varphi := (\neg\varphi) \not\vdash \top$.

System **DiaL** contains all propositional tautologies and all instantiations of the following:

- K** $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi),$
C $((\varphi \not\vdash \chi) \wedge (\psi \not\vdash \chi)) \rightarrow ((\varphi \vee \psi) \not\vdash \chi).$

Rules of inference are:

- *modus ponens*: from $\varphi \rightarrow \psi$ and φ conclude ψ ,
- *necessitation*: from φ conclude $\Box\varphi$,
- **R1**: from $\varphi \rightarrow \psi$ conclude $(\chi \not\vdash \psi) \rightarrow (\chi \not\vdash \varphi),$
- **R2**: from $\varphi \rightarrow \psi$ conclude $(\psi \not\vdash \chi) \rightarrow (\varphi \not\vdash \chi).$

Proof theory:

- theorems: $\vdash_{\mathbf{DiaL}} \varphi$,
- derivations: $\Gamma \vdash_{\mathbf{DiaL}} \varphi$

A derivation from Γ may use theorems of **DiaL**, assumptions from Γ , and modus ponens.

Any logic that extends **DiaL** is a *dialectical logic*.

Are there examples of dialectical logics? Yes!

- modal logics: $\varphi \not\leq \psi := \Box(\neg\varphi \vee \neg\psi)$,
- interpretability logics: $\varphi \not\leq \psi := \varphi \triangleright (\neg\psi)$,
- (some?) Heyting-Lewis logics: $\varphi \not\leq \psi := \psi \rightarrow (\neg\varphi)$,

Thus, several familiar classes of logics can be viewed as dialectical logics. And we can use them in order to obtain semantics for our logics.

Definition

A *dialectical frame* $\mathfrak{F}$ is a triple (W, R, S) , where W is a non-empty set, R a binary relation on W and S a ternary relation on W .

Definition

A *dialectical model* is a pair $\mathfrak{M} = (\mathfrak{F}, V)$, where $\mathfrak{F}$ is a dialectical frame and $V : \text{Prop} \rightarrow \mathcal{P}(W)$ is a *valuation* function.

Valuation induces a *forcing* relation $\Vdash$, in the following way:

$$\begin{aligned}w \Vdash p & \iff w \in V(p), \\w \Vdash \perp & \text{ for no } w \in W, \\w \Vdash \varphi \rightarrow \psi & \iff w \not\Vdash \varphi \text{ or } w \Vdash \psi, \\w \Vdash \varphi \not\vdash \psi & \iff \forall u (wRu \ \& \ u \Vdash \varphi \Rightarrow \exists v (S(w, u, v) \ \& \ v \Vdash \neg\psi)).\end{aligned}$$

We also write $\mathfrak{M}, w \Vdash \varphi$, if we want to specify the model $\mathfrak{M}$.

Forcing relation extends the valuation function to a set of all formulae:

$$V(\varphi) = \{w \in W : w \Vdash \varphi\}.$$

The proof system captures exactly the formulas valid on all dialectical frames.

Theorem (Strong completeness)

The following are equivalent:

- $\Gamma \vdash_{\text{DiaL}} \varphi$
- for every dialectical model $\mathfrak{M}$ and every w , if $\mathfrak{M}, w \Vdash \Gamma$, then $\mathfrak{M}, w \Vdash \varphi$.

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This logic formalizes the notion of dialectics, which is often criticized for its lack of rigour and vagueness (from Hegelian and Marxist traditions in particular).

Consider this sentence from the work of Etienne Balibar: “This is precisely the first meaning we can give to the idea of dialectic: a logic or form of explanation specifically adapted to the determinant intervention of class struggle in the very fabric of history” (The Philosophy of Marx [New York: Verso, 1995]). If you read that sentence quickly, it can sound pretty good. The remedy is to read it more slowly, and you will then recognize it to be a wonderful paradigm of bullshit: yet I know Balibar to be an honest thinker.
– G. A. Cohen: Deeper into Bullshit

(G. A. Cohen defines bullshit as a product of discourse featuring unclarifiable unclarity)

The system **DiaL** allows us to study different dialectical theories by adding principles to our axiom schemata, which might correspond to various philosophers.

The question: Which principles correspond to which philosophers?

Introduction
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Dialectical logic
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Potential implications and applications
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Thank you!